Experimental Investigation of the Ratio of the Circumference and the Diameter of Several Objects with a Circular Cross-section *

Ima Fizikist

October 31, 2023

Abstract

In this investigation, we examined the hypothesis that a circle's circumference, C, and diameter, D, are directly proportional. We measured the circumference and diameter of five circular objects ranging from 2 cm to 7 cm in diameter. Vernier calipers or a ruler were used to measure the diameter of each object, and a piece of paper was wrapped around each cylinder to determine its circumference. Numerical analysis of these circular objects yielded the unitless C/D ratio of 3.14 ± 0.03, which agrees with the accepted value of π. Graphical analysis led to a less precise but equivalent estimate of 3.15 ± 0.11 for this ratio. These results support commonly accepted geometrical theory, which states that $C = \pi D$ for all circles. However, only a narrow range of circle sizes were analyzed, so additional data should be taken to investigate whether the constant ratio hypothesis applies to very large and very small circles.

1 Introduction

1.1 Objective

Theoretically, the circumference, C, of a circle is equal to πD, where D is the circle's diameter. The goal of this project is to measure C and D of several objects to test the hypothesis that

$$C = \pi D. \tag{1}$$

*This sample lab report was adapted from the University of North Carolina website `http://physics.unc.edu/undergraduate-program/course-offerings/labs/sample-report/`

Table 1: Table of measurements of the circumference and diameter of five objects.

Object Description	D (cm)	C (cm)	$r = D/C$	Measuring Device
Penny coin	1.90	5.93	3.12	Vernier caliper, paper
“D” cell battery	3.30	10.45	3.17	Vernier caliper, paper
PVC cylinder A	4.23	13.30	3.14	Vernier caliper, paper
PVC cylinder B	6.04	18.45	3.06	Plastic ruler, paper
Tomato soup can	6.6	21.2	3.21	Plastic ruler, paper

1.2 Procedure

Five objects were chosen so that their circumference and diameter measurements could be obtained easily and reproducibly. Therefore, we did not use irregularly shaped objects or ones that could be deformed when measured. The diameter of each of the five objects was measured with either a plastic ruler with a 1 mm resolution or a vernier caliper with a 0.05 mm resolution. The circumference and diameter of each object were measured with the same measuring device in case the two instruments did not have the same calibration. The circumference measurement was obtained by tightly wrapping a small piece of paper around the object, marking the circumference on the paper with a pencil, and measuring this distance with a ruler or caliper. Table 1 summarizes the measurements and measuring devices used.

2 Analysis

The measured ratios C/D are also summarized in Table 1. The average ratio is $C/D = 3.14 \pm 0.03$. The uncertainty associated with the average C/D ratio is the standard error of the five C/D values, which is equal to the standard deviation (0.06) divided by the square root of N, which in this case is 5 since there were five measurements. From this empirical investigation, the average C/D ratio agrees with the accepted value of π (3.1415926...).

Another way to visualize and calculate this constant circle ratio is by graphing the circumference versus diameter for each object (see Figure 1). Graphs are especially useful for examining possible trends over the range of measurements.

If C is proportional to D, we should get a straight line through the origin.

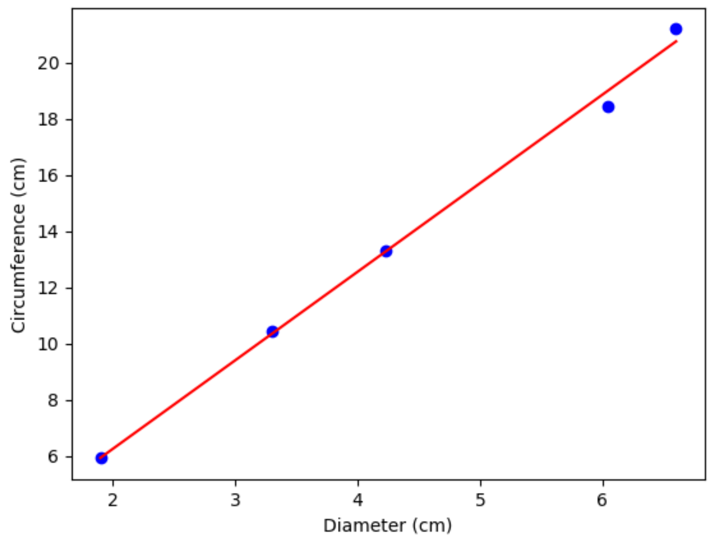

Figure 1: Plot of circumference versus diameter for five different objects.

From our numerical results, we would expect the slope of the C vs. D graph to be equal to π. The slope of the best-fit line is 3.15 ± 0.11, which is equal to π within the slope's estimated uncertainty. As expected, the intercept is essentially zero: (0.0 ± 0.5) cm.

3 Discussion

Our results support the original hypothesis for 5 circles ranging in size from 2 cm to 7 cm in diameter. The C/D ratio for our objects is essentially constant (3.14 ± 0.03) and equal to π. The specified uncertainty is the standard error of the C/D ratio for the five objects. Graphical analysis also supports the "directly proportional" hypothesis. The line has an intercept (0.0 ± 0.5) cm which is equal to zero within the estimated uncertainty and a slope 3.15 $\pm$ 0.11 which agrees with the accepted value of π . The larger uncertainty from the graphical analysis suggests that the random measurement errors may be larger than estimated in the numerical analysis. A more extensive investigation of this C/D relationship over a wider range of circle sizes should be performed to verify that this ratio is indeed constant for all circles.

The uncertainty in the measurements could be due to the paper-wrapping method of measuring the circumference, circles that may not be perfect, and the limited precision of the measuring devices. The use of paper to measure the circumference was the most significant source of uncertainty. It is unlikely, however, that this measurement technique biased our results since the technique probably gave measurements of C that were too high in some cases and too low in others.

The C/D ratio for a perfect circle was defined long ago by the Greek symbol: $\pi = 3.14159....$ Our measured value appears to be consistent with the accepted value of π within the limits of our experimental uncertainty. This unique C/D ratio has many important applications wherever circles or spheres are encountered. More information about π can be found in the book *A History of Pi.*[1]

References

[1] P. Beckmann, *A History of Pi*, (Griffin, 1971).