Consumer's Problem:

1. We know that whenever \( u(c, c') = \ln c + \beta \ln c' \) and \( we \equiv y - t + \frac{y'}{1+r} - \frac{c'}{1+r} = c + \frac{c'}{1+r} \), the solution to the consumer problem is given by: 
   \[ c^* = \frac{1}{1+\beta} we, \quad c'^* = \frac{\beta(1+r)}{1+\beta} we, \quad s^* = y - t - c^*. \]
   Now assume that \( \beta = 0.5, r = 0.1, y = 10, t = 2, y' = 12, \) and \( t' = 3 \) and answer the following questions. For every numerical answer use only 2 decimal places and for every graph make sure to show where the endowment point is.

   (a) What is the solution to the consumer problem? Graph this solution on the \((c, c')\) space.

   (b) Assume that in period 1, \( y \) increases to 20. Re-solve the problem and show the effect of this increase in \( y \) on a graph. Make sure to explain what happens to \( c, c', \) and \( s \).

   (c) Now assume that \( y' \) increases to 22, but \( y \) is again 10. Re-solve the problem and show the effect of this increase in \( y' \) on a graph. Make sure to explain what happens to \( c, c', \) and \( s \).

   (d) From parts (a), (b), and (c), is this consumer a lender or a borrower?

   (e) Now analyze the effects on \( c, c', \) and \( s \) of simultaneous increases in \( y \) to 20 and \( y' \) to 22. Graph your solution and compare it to your solutions to (b) and (c).

   (f) How would your solution to the original problem change if the government decided to increase period 1 taxes by 50%. Show this effect on a graph.

   (g) What if the government decides to permanently increase taxes by 50%? Compare your answer here to your answer from (f) with the help of a graph.

   (h) Reset all variables to their original values and re-solve the problem for a new interest rate of 0.2. Show the effect of this change on a graph. Are these solutions consistent with the model’s predictions (see tables on page 317)? You need to comment on the substitution and income effects.

2. Re-work all of the problems above for \( \beta = 2 \). Explain why this change in \( \beta \) has caused your answers to change (use economics here, not math).

Competitive Equilibrium and Ricardian Equivalence:

3. Assume there are 100 identical consumers in the economy and that each individual’s current income \((y)\) is 20, future income \((y')\) is 24, the interest rate \((r)\) is 10%, and taxes are 4 currently and 6 in the future.
(a) What is each consumer’s lifetime wealth \( (w_e) \)?

(b) Calculate each consumer’s endowment points.

(c) Assume that given taxes, income, and interest rates, each individual decides to consume 12 units in the present \( (c = 12) \). Calculate future consumption \( (c') \) and current savings \( (s) \). Show this solution on the \((c, c')\) space.

(d) What are aggregate private savings \( (S_p) \) in this economy?

(e) What are aggregate current taxes \( (T) \) and aggregate future taxes \( (T') \)?

(f) Calculate aggregate present consumption \( (C) \) and aggregate present output \( (Y) \).

(g) From (f) what must be present government spending \( G \)?

(h) Are credit markets clearing in this economy? That is, is \( S_p = B \)?

(i) Using your answers in (e) and (g), calculate government expenditures in the future \( (G') \) using the intertemporal government budget constraint equation.

(j) Is this economy in equilibrium? Explain.

(k) Now assume that the government cuts period 1 taxes in half. What must be taxes in period 2 in order to guarantee that the government’s intertemporal budget is satisfied assuming \( G \) and \( G' \) are exogenous.

(l) How much does the government need to borrow in period one in order to finance its expenditures \( G \)?

(m) What is the effect of this change in the timing of taxes on the individual lifetime wealth?

(n) Calculate each individual’s endowment points? Compare your answer with part (b)?

(o) What will be the consumer’s allocations in terms of consumption today \( (c) \), consumption tomorrow \( (c') \) and savings \( (s) \)?

(p) Using only one graph, combine your solutions from part (c) with your solutions from part (o). Make sure to also graph the endowment points.

(q) Are credit markets clearing, that is, \( S_p + S_g = 0 \), where \(-S_g = B\)? Explain.

(r) From this exercise, is it possible to verify that the Ricardian equivalence theorem holds for this economy? Explain this by comparing the consumer’s behavior before and after the
change in tax policy.

4. Re-work all of the problems above for a doubling of taxes in the current period. That is, in part (k), the government doubles taxes in period 1.