Let's color a 2 by 2 chessboard with 3 colors; which we will call red, blue or green. Since there are four squares, we could distribute the colors as $(4,0,0)$, $(3,1,0)$, $(2,2,0)$, or $(2,1,1)$.

For the one color $(4,0,0)$ case, there are clearly 3 cases: the red board, the blue board and the green board.

For the $(3,1,0)$ case consider we should choose the 2 colors we'll use. There are $\binom{3}{2} = 3$ ways to do this. But then for each color choice there are 2 colorings $(3+1$ or $1+3)$. So we have 6 colorings in this case.

For the $(2,2,0)$ case, we first choose the two colors we use. For each choice there are two distinct colorings, the diagonal pattern and the adjacent pattern. We thus again have 6 colorings.

For the $(2,1,1)$ case, we need only choose the color used twice. There are 3 ways to do this. But then for each such case we get two colorings, depending on whether the color used twice colors adjacent or diagonally opposite squares. So we have 6 more colorings here.

This leaves $3+6+6+6=21$ colorings altogether.