Let's color a 2 by 2 chessboard with 3 colors; which we will call red, blue or green. Since there are four squares, we could distribute the colors as (4, 0, 0), (3, 1, 0), (2, 2, 0), or (2, 1, 1).

For the one color (4, 0, 0) case, there are clearly 3 cases: the red board, the blue board and the green board.

For the (3, 1, 0) case consider we should choose the 2 colors we'll use. There are \( \binom{3}{2} = 3 \) ways to do this. But then for each color choice there are 2 colorings (3 + 1 or 1 + 3). So we have 6 colorings in this case.

For the (2, 2, 0) case, we first choose the two colors we use. For each choice there are two distinct colorings, the diagonal pattern and the adjacent pattern. We thus again have 6 colorings.

For the (2, 1, 1) case, we need only choose the color used twice. There are 3 ways to do this. But then for each such case we get two colorings, depending on whether the color used twice colors adjacent or diagonally opposite squares. So we have 6 more colorings here.

This leaves 3 + 6 + 6 + 6 = 21 colorings altogether.