MT Review
Even Answers

Chapter 1 Supp. Ex. (p88-90)

18. Suppose c_1 and c_2 are constants such that

$$c_1\mathbf{v}_1 + c_2(\mathbf{v}_1 + \mathbf{v}_2) = \mathbf{0} \qquad (*)$$

Then $(c_1 + c_2)\mathbf{v}_1 + c_2\mathbf{v}_2 = \mathbf{0}$. Since $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent, both $c_1 + c_2 = 0$ and $c_2 = 0$. It follows that both c_1 and c_2 in $(*)$ must be zero, which shows that $\{\mathbf{v}_1, \mathbf{v}_1 + \mathbf{v}_2\}$ is linearly independent.

20. If $T(\mathbf{u}) = \mathbf{v}$, then, since T is linear, $T(-\mathbf{u}) = T((-1)\mathbf{u}) = (-1)T(\mathbf{u}) = -\mathbf{v}$.

Chapter 2 Supp Ex (p160-161)

8. By definition of matrix multiplication, the matrix A satisfies

$$A\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$$

Right-multiply both sides by the inverse of $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$:

$$A = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 4 & -1 \end{bmatrix}$$

10. Since A is invertible, so is A^T, by the Invertible Matrix Theorem. Then A^TA is the product of invertible matrices and so is invertible. Thus, the formula $(A^TA)^{-1}A^T$ makes sense. By Theorem 6 in Section 2.2,

$$(A^TA)^{-1} \cdot A^T = A^{-1}(A^T)^{-1}A^T = A^{-1}I = A^{-1}$$

An alternative calculation:
$(A^TA)^{-1}A^T \cdot A = (A^TA)^{-1}(A^TA) = I$. Since A is invertible, this equation shows that its inverse is $(A^TA)^{-1}A^T$.

18. Suppose $\mathbf{x}$ satisfies $A\mathbf{x} = \mathbf{b}$. Then $CA\mathbf{x} = C\mathbf{b}$. Since $CA = I$, $\mathbf{x}$ must be $C\mathbf{b}$. This shows that $C\mathbf{b}$ is the only solution of $A\mathbf{x} = \mathbf{b}$.