

Final Review Answers

(to Evers)

Ch 4 supp p 222 #2) Any two of the three $\begin{bmatrix} 1 \\ 2 \\ -1 \\ 2 \end{bmatrix}$ $\begin{bmatrix} -2 \\ 5 \\ -4 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 5 \\ -8 \\ 7 \\ 1 \end{bmatrix}$

#4) The vectors f and g are not SCALAR multiples of each other, so $\{f, g\}$ is linearly independent.

p 238 #30) The equation $Ax = b$ is consistent if and only if $\text{rank}[A \ b] = \text{rank } A$ because the two ranks are equal if and only if b is not a pivot column of $[A \ b]$.

p 229 #24) $[P]_B = (6, 3, -2)$

p 326 #2) Suppose $B\vec{x} \neq 0$ and $AB\vec{x} = \lambda\vec{x}$ for some λ . Then $A(B\vec{x}) = \lambda\vec{x}$. Left multiply each side by B , and obtain $BA(B\vec{x}) = \lambda B\vec{x}$. This equation says $B\vec{x}$ is an eigenvector of BA . (since $B\vec{x} \neq 0$).

18. The eigenvalues of A are 1 and .6. Use this to factor A and A^k .

p 326 #18)

$$A = \begin{bmatrix} -1 & -3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & .6 \end{bmatrix} \cdot \frac{1}{4} \begin{bmatrix} 2 & 3 \\ -2 & -1 \end{bmatrix}$$

$$A^k = \begin{bmatrix} -1 & -3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1^k & 0 \\ 0 & .6^k \end{bmatrix} \cdot \frac{1}{4} \begin{bmatrix} 2 & 3 \\ -2 & -1 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} -1 & -3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -2 \cdot (.6)^k & -(.6)^k \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} -2 + 6(.6)^k & -3 + 3(.6)^k \\ 4 - 4(.6)^k & 6 - 2(.6)^k \end{bmatrix}$$

$$\rightarrow \frac{1}{4} \begin{bmatrix} -2 & -3 \\ 4 & 6 \end{bmatrix} \text{ as } k \rightarrow \infty$$

S.6 #17a) $A = \begin{bmatrix} 0 & 1.6 \\ 1.3 & .8 \end{bmatrix}$ b) evaluate 1, 2 with evector $\begin{bmatrix} 4 \\ 3 \end{bmatrix} = v_1$
 $\rightarrow .4$ $\begin{bmatrix} 4 \\ -1 \end{bmatrix} = v_2$

$$x_k = c_1 (1.2)^k v_1 + c_2 (.4)^k v_2 \quad (\text{where } \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \{v_1, v_2\})$$

Population grows by factor of 1.2 each year
 eventual ratio of juvenile to adult is 4 to 3