

11) alternating signs, $\frac{1}{(n+1)\ln(n+1)} < \frac{1}{n\ln(n)}$ $\frac{1}{n\ln(n)} \rightarrow 0$ so converges by Leibniz. $\sum \frac{1}{n\ln(n)}$ diverges by integral test since $\int_2^{\infty} \frac{1}{x\ln x} = \ln|\ln x| \Big|_2^{\infty} \rightarrow \infty$ so converges conditionally.

13) $\int_2^{\infty} \frac{2}{x(\ln x)^3} dx$ $u = \ln x$ $du = \frac{1}{x} dx$ $= \frac{-2}{2u^2} \Big|_{x=2}^{x=\infty} = \frac{-1}{(\ln x)^2} \Big|_2^{\infty}$ converges

so converges by integral test
19) Ratio test $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{(n+1)!}{2 \cdot 5 \cdot 8 \cdots (3n+2)(3n+5)} \cdot \frac{2 \cdot 5 \cdot 8 \cdots (3n+2)}{n!} = \lim_{n \rightarrow \infty} \frac{n+1}{3n+5} = \frac{1}{3} < 1$ so converges

34) Root test $\lim_{n \rightarrow \infty} \frac{2n}{n^2} = 0 < 1$ so converges

20) alternating signs, $\frac{n+1}{(n+2)(n+3)} < \frac{n}{(n+1)(n+2)}$ since $(n+1)^2 < n^2 + 3n$ since $1 < n$ so decreasing, $\frac{n}{(n+1)(n+2)} \rightarrow 0$ by L'Hospital so converges by Leibniz
limit comparison $\lim_{n \rightarrow \infty} \frac{\frac{n}{(n+1)(n+2)}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2 + 3n + 2} = 1$ by L'Hospital
so does not converge absolutely so converges conditionally

Exercises

1-38 □ Test the series for convergence or divergence.

1. $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + n}$ D

2. $\sum_{n=1}^{\infty} \frac{n-1}{n^2 + n}$ D

3. $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$ C

4. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n-1}{n^2 + n}$ CC

5. $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$ CA

~~6. $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$ C~~

7. $\sum_{k=1}^{\infty} k^{-1.7}$ C

8. $\sum_{n=0}^{\infty} \frac{10^n}{n!}$ C

9. $\sum_{n=1}^{\infty} \frac{n}{e^n}$ C

10. $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ C

11. $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln n}$ CC

12. $\sum_{n=1}^{\infty} \sin n$ D

13. $\sum_{n=2}^{\infty} \frac{2}{n(\ln n)^3}$ C

14. $\sum_{n=1}^{\infty} \frac{n^2 + 1}{n^3 + 1}$ D

15. $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$ C

16. $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n} - 1}$ CC

17. $\sum_{n=1}^{\infty} \frac{3^n}{5^n + n}$ C

18. $\sum_{k=1}^{\infty} \frac{k+5}{5^k}$ C

19. $\sum_{n=0}^{\infty} \frac{n!}{2 \cdot 5 \cdot 8 \cdots (3n+2)}$ C

21. $\sum_{i=1}^{\infty} \frac{1}{\sqrt{i(i+1)}}$ D

23. $\sum_{n=1}^{\infty} (-1)^n 2^{1/n}$ D

25. $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$ CC

27. $\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$ CA

29. $\sum_{k=1}^{\infty} \frac{k \ln k}{(k+1)^3}$ C

31. $\sum_{n=1}^{\infty} \frac{2^n}{(2n+1)!}$ C

33. $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n\sqrt{n}}$ C

~~35. $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$ C~~

37. $\sum_{n=1}^{\infty} (\sqrt{2} - 1)^n$ C

20. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{(n+1)(n+2)}$ CC

22. $\sum_{n=1}^{\infty} \frac{\sqrt{n^2 - 1}}{n^3 + 2n^2 + 5}$ C

24. $\sum_{n=1}^{\infty} \frac{\cos(n/2)}{n^2 + 4n}$ CA

~~26. $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$ C~~

28. $\sum_{n=1}^{\infty} \frac{n^2 + 1}{5^n}$ C

30. $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$ C

32. $\sum_{j=1}^{\infty} (-1)^j \frac{\sqrt{j}}{j+5}$ CC

34. $\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$ C

~~36. $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$ C~~

~~38. $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$ C~~