Block 8, 2009 REVIEW ANSWERS

1a) $2\sum_{n=1}^{\infty} \frac{1}{n^{5/6}}$ $p<1$ so $\boxed{\text{diverges}}$ (could check with integral test)

b) Ratio test $\lim_{n\to\infty} \frac{\cancel{(n+1)!}^{(n+1)} \cdot \cancel{3^{n+1}}^{3}}{(3n+3)(3n+2)(3n+1)\cancel{(3n)!}} \cdot \frac{\cancel{(3n)!}}{\cancel{n!}\cdot\cancel{3^n}}$

$= \lim_{n\to\infty} \frac{3n+3}{(3n+3)(3n+2)(3n+1)} = 0 < 1$ $\boxed{\text{converges}}$

c) $\frac{1}{n^3+n} < \frac{1}{n^3}$ which $\boxed{\text{converges}}$ since $p>1$

d) signs alternate, $\frac{1}{\sqrt[3]{5n^2+n}} \to 0$, $\frac{1}{\sqrt[3]{5(n+1)^2+n+1}} < \frac{1}{\sqrt[3]{5n^2+n}}$ so converges by Leibniz
abs. value diverges by limit comparison to $\frac{1}{n^{2/3}}$
so $\boxed{\text{converges conditionally}}$

2a) If $y(t)$ = amount of money owed after t years
then $\boxed{\frac{dy}{dt} = .139y - 4200 \;;\; y(0) = 14688}$

b) $\int \frac{dy}{.139y-4200} = \int dt$

$\frac{1}{.139} \ln|.139y - 4200| = t + c$

$y = c' e^{.139t} + 30{,}215.83$

$14688 = c' + 30{,}215.83$ so $c' = -15{,}527.83$

$y = -15{,}527.83 e^{.139t} + 30{,}215.83$

$0 = -15{,}527.83 e^{.139t} + 30{,}215.83$

$.6657 = .139t$

$\sim \boxed{4.8 \text{ years}}$

c) $y = ke^{.139t} + 30{,}215.83$

$0 = ke^{.139(3)} + 30215.83$

$k = -19{,}912.86$

at $t=0$, $y = 10{,}302.97$

so down payment $= 14688 - 10302.97 = \boxed{\$4385.03}$

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3a) $\int_0^1 e^{-x^2}\,dx \approx \int_0^1 1-x^2+\frac{x^4}{2}\,dx = x-\frac{x^3}{3}+\frac{x^5}{10}\Big]_0^1 \approx \boxed{.77}$

b) $\int_{-\infty}^{\infty} 2\pi x e^{-x^2}\,dx$ $\quad u=e^{-x^2}$, $du=-2x\,dx$ $\quad = \int_{x=-\infty}^{x=\infty} -\pi e^{u}\,du = -\pi e^{-x^2}\Big]_0^{\infty}$

$\boxed{=\pi}$

c) $M_y = \rho\int_0^1 x e^{-x^2}\,dx = -\frac{\rho}{2}\int_{x=0}^{x=1} e^{u}\,du = -\frac{\rho}{2}e^{-x^2}\Big]_0^1 = \frac{\rho}{2}-\frac{\rho}{2e}$

$= .316\rho$

$\bar{x} = \frac{M_y}{\rho(\text{area})} = \frac{.316\rho}{.77\rho} = \boxed{.41}$

4a) $\frac{3}{\sqrt[4]{x^2-1}} > \frac{3}{x^{1/2}}$ and $\lim_{b\to\infty}\int_1^b \frac{3}{x^{1/2}}\,dx = \lim_{b\to\infty} 6\sqrt{b}-6=\infty$

so $\boxed{\text{diverges}}$

b) Let $u=2x-4$, $du=2dx$ get $\frac{5}{2}\int u^{-1/4}\,du = \frac{10}{3}\left[4^{3/4}-0\right] = \boxed{9.43}$

4c) $\int \frac{(\ln x)^2}{x^2}\,dx$ $\quad u=(\ln x)^2$, $u'=2\ln(x)(\frac{1}{x})$ $\quad v'=x^{-2}$, $v=-\frac{1}{x}$

$= -\frac{(\ln x)^2}{x} + 2\int\frac{(\ln x)}{x^2}\,dx$ $\quad u=\ln x$, $u'=\frac{1}{x}$ $\quad v'=x^{-2}$, $v=-\frac{1}{x}$

$= \frac{-(\ln x)^2}{x} - 2\frac{\ln x}{x} + 2\int\frac{1}{x^2}\,dx = \boxed{\frac{-(\ln x)^2}{x} - \frac{2\ln x}{x} - \frac{1}{x} + c}$

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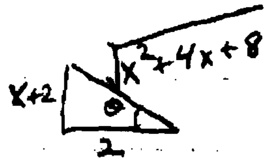
4 d) $\int_1^2 \frac{5x}{(x+2)^2+4}\,dx$ $\quad x+2 = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$



$$= \int \frac{5(\cancel{2}\tan\theta - \cancel{2})\cancel{2}\cancel{\sec^2\theta}\, d\theta}{\cancel{4}\cancel{\sec^2\theta}} = \int 5\tan\theta - 5\, d\theta = 5\ln|\sec\theta| - 5\theta$$

$$= 5\ln\frac{\sqrt{x^2+4x+8}}{2} - 5\tan^{-1}\frac{x+2}{2}\Big]_1^2$$

$$= \boxed{.455}$$

5 a) $f(x) = \ln(2x+1)$ $\quad f(0) = 0$

$f'(x) = \frac{2}{2x+1}$ $\quad f'(0) = 2$

$f''(x) = -(2x+1)^{-2}(4)$ $\quad f''(0) = -4$

$f'''(x) = 2(2x+1)^{-3}(8)$ $\quad f'''(0) = 16$

$f^{iv}(x) = -6(2x+1)^{-4}(16)$ $\quad f^{iv}(0) = -6\cdot 16$

Taylor series is $\boxed{2x - 2x^2 + \frac{8x^3}{3} - \frac{16x^4}{4} + \cdots = \sum_{n=1}^{\infty} \frac{(-2)^n x^n}{n}}$

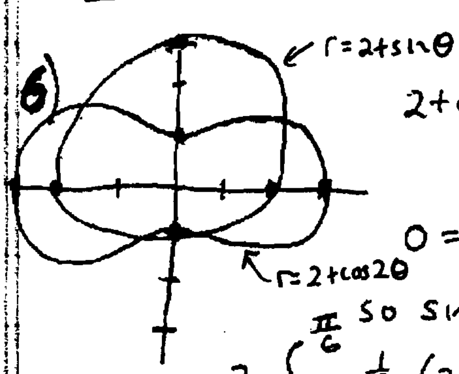
b) $\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{2^{n+1}|x|^{n+1}}{n+1}}{\frac{2^n|x|^n}{n}} \to 2|x|$

so converges if $|x| < \frac{1}{2}$.

diverges if $|x| > \frac{1}{2}$.

$\boxed{\text{Radius of conv} = \frac{1}{2}}$

6)



$2+\cos2\theta = 2+\sin\theta$

$1 - 2\sin^2\theta = \sin\theta$

$0 = 2\sin^2\theta + \sin\theta - 1 = (2\sin\theta - 1)(\sin\theta + 1)$

so $\sin\theta = \frac{1}{2}$ or -1 so $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$.

$$2\int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} \frac{1}{2}(2+\cos2\theta)^2 - \frac{1}{2}(2+\sin\theta)^2\, d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\pi/6} 4\cos2\theta + \cos^2 2\theta - 4\sin\theta - \sin^2\theta\, d\theta$$

$$= 2\sin2\theta + \cancel{\tfrac{1}{2}\theta} + \frac{1}{8}\sin4\theta + 4\cos\theta - \cancel{\tfrac{1}{2}\theta} + \frac{1}{4}\sin2\theta\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{6}}$$

$$= \frac{9}{4}\sin\frac{\pi}{3} + \frac{1}{8}\sin\frac{2\pi}{3} + 4\cos\frac{\pi}{6} = \boxed{\frac{51\sqrt{3}}{16}}$$