MT Answers

4 a) $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\cdots = 2$ (geometric converges)

b) $1+2+4+8+\cdots$ (geometric diverges)

c) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$ (diverges but terms $\to 0$)

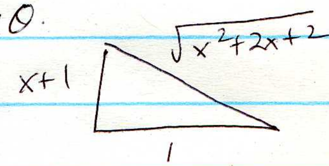
d) $1, \; 3/2, \; 7/4, \; \ldots, \; 2-\frac{1}{2^n}, \ldots$ (sequence $\to 2$)

e) $\frac{1}{1-r} = 3$ if $r = 2/3$ so $1+\frac{2}{3}+\left(\frac{2}{3}\right)^2+\cdots$ (series $\to 3$)

f) $\int_0^1 \frac{1}{x}\,dx$ (improper w/o ∞, diverges)

5a) $\int \frac{1}{\left((x+1)^2+1\right)^{3/2}}$ let $x+1=\tan\theta$, $dx = \sec^2\theta\, d\theta$

$\int \frac{\sec^2\theta}{\sec^3\theta}\,d\theta = \int \cos\theta\, d\theta$



$= \sin\theta + c$

$= \frac{x+1}{\sqrt{x^2+2x+2}} + c$

b) $\int_3^\infty \frac{1}{x(2-x)}\,dx$

$\frac{1}{x(2-x)} = \frac{A}{x} + \frac{B}{2-x}$

$A(2-x) + Bx = 1$ so $A=\frac{1}{2}$ $B=\frac{1}{2}$

$= \lim_{R\to\infty} \left.\frac{1}{2}\ln|x| - \frac{1}{2}\ln|2-x|\right]_3^R$

$= \lim_{R\to\infty} \frac{1}{2}\ln\left|\frac{R}{2-R}\right| - \frac{1}{2}\ln 3 = \boxed{-\frac{1}{2}\ln 3}$

c) $\int x\cos^2 x\,dx$ $u = x$ $v' = \cos^2 x$

$u' = 1$ $v = \int \frac{1+\cos 2x}{2}\,dx = \frac{1}{2}x + \frac{1}{4}\sin 2x$

$\frac{1}{2}x^2 + \frac{1}{4}x\sin 2x - \int \frac{1}{2}x + \frac{1}{4}\sin 2x\,dx$

$= \boxed{\frac{1}{4}x^2 + \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + c}$

d) $\int \frac{\sec^2 x}{1-\tan x}$ $u = 1-\tan x$, $du = -\sec^2 x\,dx$

$= -\int \frac{1}{u}\,du = -\ln|1-\tan x| + c$