MT Answers

20 pts

1 a) $5 + \frac{10}{3} + \frac{20}{9} + \frac{40}{27} + \cdots$

geometric, $r = \frac{2}{3}$, converges to $\frac{5}{1-2/3} = 15$

b) $\sum_{n=1}^{\infty} \frac{5^n}{n^5}$ diverges since n^{th} term $\to \infty$

c) $\sum_{n=2}^{\infty} \frac{2}{n(\ln n)^2}$ integral test $\int \frac{2}{x(\ln x)^2}$ $u = \ln x$, $du = \frac{1}{x}dx$

$= \lim_{R\to\infty} \frac{-2}{\ln R} + \frac{2}{\ln 2}$ converges

d) $\sum_{n=1}^{\infty} \frac{3}{n\sqrt{5n+1}}$ compare to $\frac{3}{\sqrt{5}n^{3/2}}$

$\frac{3}{n\sqrt{5n+1}} < \frac{3}{\sqrt{5}n^{3/2}}$ converges $p = 3/2$

10 pts

2) volume of region bounded by $y = x$ and $x^2 + y^2 = 1$ and y axis revolved around y axis

$$= \int_0^{1/\sqrt{2}} 2\pi x\left(\sqrt{1-x^2} - x\right) dx$$

$$= \int_0^{1/\sqrt{2}} 2\pi x\sqrt{1-x^2}\,dx - \int_0^{1/\sqrt{2}} 2\pi x^2 dx$$

$u = 1 - x^2$, $du = -2x\,dx$

$$= -\pi \int_{x=0}^{x=1/\sqrt{2}} u^{1/2}\,du - \frac{2\pi}{6\sqrt{2}}$$

$$= -\frac{2\pi}{3}\left(1-x^2\right)^{3/2}\Big]_0^{1/\sqrt{2}} - \frac{2\pi}{6\sqrt{2}}$$

$$= -\frac{\pi\sqrt{2}}{3} + \frac{2\pi}{3} = .613$$

10 pt

3) Slice of water at height y must be lifted $3-y$ ft

has force $62.5\left(\frac{2y}{\sqrt{3}}\right)(6)$

Work done pumping water out of trough 6 ft long, 3 ft deep, ends are equilateral triangles. (water weighs 62.5 lb/ft³)

Work $= 4\sqrt{3}(62.5)\int_0^3 3y - y^2\,dy = 4\sqrt{3}(62.5)\frac{9}{2} =$ **1948 ft-lb**