

## FINAL REVIEW

The exam will be available by 8AM, and due at noon. You will be allowed to use the one page of tables given to you for previous quizzes, and the one page of notes you prepared for yourself in advance. You may not use any other books or notes, nor any help from other people. You may use the arithmetic, trig and inverse trig, and exponential and log functions and  $y=f(x)$  graphs only on your calculator or computer—no polar graphing, calculus, or programmable features.

1. Determine whether each of the following series converges or diverges, absolutely or conditionally, and give a convincing reason why.

a.  $2 + \frac{2}{\sqrt[6]{2^5}} + \frac{2}{\sqrt[6]{3^5}} + \frac{2}{\sqrt[6]{4^5}} \cdots$

b.  $\sum_{n=1}^{\infty} \frac{n!3^n}{(3n)!}$

c.  $\sum_{n=1}^{\infty} \frac{1}{n^3+n}$

d.  $\sum \frac{(-1)^n}{\sqrt[3]{5n^2+n}}$

2. You decide to buy a 2004 Accord for \$14,688. The car dealership offers you a loan at 13.9% interest, no down payment, which you must pay off in monthly payments of \$350.

a. Assume that interest is compounded continuously and that the sum of the 12 monthly payments is paid out continuously throughout the year. Write down a differential equation for the amount of money still owed after  $t$  years.

b. Solve the differential equation. How long does it take to pay off the loan?

c. With the same monthly payments, and the same interest rate, how much down payment would you need to be able to pay the loan off in 3 years instead?

3. Consider the region enclosed between the bell-shaped curve  $y = e^{-x^2}$ , the  $y$  axis and the  $x$  axis.

a. Write an integral equal to the area of this region between  $x=0$  and  $x=1$ , and approximate it using the fourth Taylor polynomial for  $y = e^{-x^2}$ .

b. Find the volume produced when the whole region from  $x=0$  to  $x = \infty$  is rotated around the  $y$ -axis.

c. Find the  $x$  component of the center of mass of a thin plate of uniform density occupying the region between  $x=0$  and  $x=1$ .

4. Evaluate the following integrals:

a.  $\int_1^{\infty} \frac{3}{\sqrt{x^2-1}} dx$

b.  $\int_2^4 \frac{5}{\sqrt[3]{2x-4}} dx$

c.  $\int \frac{(\ln x)^2}{x^2} dx$

d.  $\int_1^2 \frac{5x}{x^2+4x+8} dx$

5. a. Find the Taylor series expansion for the function  $f(x) = \ln(2x+1)$  centered at 0.

b. Find the radius of convergence.

6. a. Graph the two polar curves  $r = 2 + \cos 2\theta$  and  $r = 2 + \sin \theta$  without the use of your calculator.

b. Find the area inside the first graph but outside the second.