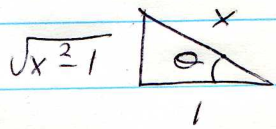
MT Review Answers

1a) $\int \frac{\sqrt{x^2-1}}{x}\,dx$ Let $x = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$= \int \frac{\tan\theta}{\cancel{\sec\theta}}\cancel{\sec\theta}\tan\theta\,d\theta$



$= \int \sec^2\theta - 1\,d\theta = \tan\theta - \theta + c$

$= \boxed{\sqrt{x^2-1} - \sec^{-1}x + c}$

1b) $\int_1^2 \frac{\ln x}{\sqrt{x}}\,dx$ $u = \ln x$ $u' = \frac{1}{x}$ $v' = x^{-1/2}$ $v = 2x^{1/2}$

$= 2\sqrt{x}\ln x\Big]_1^2 - 2\int_1^2 x^{-1/2}\,dx$

$= 2\sqrt{x}\ln x\Big]_1^2 - 4\sqrt{x}\Big]_1^2$

$= \boxed{2\sqrt{2}\ln 2 - 4\sqrt{2} + 4}$

1c) $\int \sin^7 x\,dx = \int (1-\cos^2 x)^3 \sin x\,dx$

$u = \cos x$
$du = -\sin x\,dx$

$= \int (1 - 3\cos^2 x + 3\cos^4 x - \cos^6 x)\sin x\,dx$

$= -\int 1 - 3u^2 + 3u^4 - u^6\,du$

$= -\left(u - u^3 + \frac{3}{5}u^5 - \frac{1}{7}u^7\right) + c$

$= \boxed{-\cos x + \cos^3 x - \frac{3}{5}\cos^5 x + \frac{1}{7}\cos^7 x + c}$

1d) $\int_2^\infty \frac{3}{\sqrt[4]{x^3-1}}\,dx$ $\frac{3}{\sqrt[4]{x^3-1}} > \frac{3}{\sqrt[4]{x^3}}$ since $\sqrt[4]{x^3-1} < \sqrt[4]{x^3}$

Thus $\int_2^\infty \frac{3}{\sqrt[4]{x^3-1}}\,dx$ $\boxed{\text{DIVERGES}}$ by comparison to $\int_2^\infty \frac{3}{x^{3/4}}\,dx$

$\left(p = \frac{3}{4} < 1\right)$