

# MT Review Even Answers

Q10 # 22

$$\lim_{n \rightarrow \infty} \frac{\ln(n^2+1)}{\ln(n^3+1)} = \lim_{x \rightarrow \infty} \frac{\ln(x^2+1)}{\ln(x^3+1)} = \lim_{x \rightarrow \infty} \frac{\frac{2x}{x^2+1}}{\frac{3x^2}{x^3+1}} = \lim_{x \rightarrow \infty} \frac{2x^4+2x}{3x^4+3x^2} = \boxed{\frac{2}{3}}$$

Q10 #38  $\pi\left(\frac{1}{2}\right)^2 + \pi\left(\frac{1}{8}\right)^2 + \pi\left(\frac{1}{16}\right)^2 + \dots$  geometric series

$$a = \frac{\pi}{16}, r = \frac{1}{4} \text{ so sum is } \frac{\pi}{16} \cdot \frac{4}{3} = \boxed{\frac{\pi}{12}}$$

Q10 #46 <sup>comparison</sup>  $\frac{1}{n - \ln n} > \frac{1}{n}$  diverges by  $p=1$  so diverges.

Q10 #80 ratio test  $\lim_{n \rightarrow \infty} \frac{(n+1)e^{-0.02(n+1)}}{n e^{-0.02n}} = \lim_{n \rightarrow \infty} \frac{(n+1)}{n e^{0.02}} = \frac{1}{e^{0.02}} < 1$   
converges

Q10 #128  $F(x) = \int_0^x \frac{(1 + t + \frac{t^2}{2!} + \dots) - 1}{t} dt$

$$= \int_0^x \left(1 + \frac{t}{2!} + \frac{t^2}{3!} + \dots\right) dt$$

$$= x + \frac{x^2}{2 \cdot 2!} + \frac{x^3}{3 \cdot 3!} + \dots = \boxed{\sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!}}$$

Q12 #16

- $F(0,0,0) + t(1,1,1)$  is same as  $(1,1,1) + t(1,1,1)$
- $F(0,0,0) + t(1,1,1)$  same as  $(0,0,0) + t\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$
- $F$  same example as (a)
- $F(0,0,0) + t(1,1,1)$  is parallel to  $(1,0,0) + t\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$
- If two lines are parallel their direction vectors are parallel  
by page 638, this implies  $w_1 = dw_2$

## MT Review Even Answers (cont)

Q12 #32 (b) is scalar since dot product

Q12 #52 Solving both equations for  $z$ , and setting them equal

we see line is given by  $y = \frac{-4}{3} + \frac{2}{3}x$ ,  $z = \frac{7}{3} - \frac{5}{3}x$

To put in more standard form, take  $x = t$  to get

$$r(t) = \left(0, \frac{-4}{3}, \frac{7}{3}\right) + \left(1, \frac{2}{3}, -\frac{5}{3}\right)t$$

Q14 #52  $F_x = 3x^2 - y$  c.p.  $x = \frac{1}{3}$   $y = \frac{1}{3}$

$$F_y = -x - 2y + 1 \quad x = -\frac{1}{2} \text{ not in domain}$$

$D = -6x - 1 < 0$  at  $x = \frac{1}{3}$  so saddle.

So max & min must be on boundaries

$$x = 0 \quad 0 \leq y \leq 1 \quad F(0, y) = -y^2 + y \quad \max F(0, \frac{1}{2}) = \frac{1}{4} \quad \min F(0, 0) = F(0, 1) = 0$$

$$x = 1 \quad 0 \leq y \leq 1 \quad F(1, y) = 1 - y^2 \quad \max F(1, 0) = 1 \quad \min F(1, 1) = 0$$

$$y = 0 \quad 0 \leq x \leq 1 \quad F(x, 0) = x^3 \quad \max F(1, 0) = 1 \quad \min F(0, 0) = 0$$

$$y = 1 \quad 0 \leq x \leq 1 \quad F(x, 1) = x^3 - x \quad \max F(0, 1) = F(1, 1) = 0 \quad \min F\left(\frac{1}{3}, 1\right) = \frac{-2}{3\sqrt{3}}$$