Final Review Answers

1a) $\vec{r}\,'(t) = (t, 0, t^2)$

b) $\|r'(t)\| = \sqrt{t^2+t^4} = t\sqrt{1+t^2}$

c) $\int_0^{\sqrt{3}} t\sqrt{1+t^2}\,dt = \frac{1}{3}(1+t^2)^{3/2}\Big]_0^{\sqrt{3}} = \frac{7}{3}$

d) $\dfrac{\|r'\times r''\|}{\|r'\|^3} = \dfrac{\|(t,0,t^2)\times(1,0,2t)\|}{t^3(1+t^2)^{3/2}} = \dfrac{t^2}{t^3(1+t^2)^{3/2}} = \dfrac{1}{t(1+t^2)^{3/2}}$

e) $\vec{T} = \left(\dfrac{1}{\sqrt{1+t^2}}, 0, \dfrac{t}{\sqrt{1+t^2}}\right)$

$\dfrac{d\vec{T}}{dt} = \left(-t(1+t^2)^{-3/2}, 0, (1+t^2)^{-3/2}\right)$

$\vec{N} = \dfrac{\left(-t(1+t^2)^{-3/2}, 0, (1+t^2)^{-3/2}\right)}{(1+t^2)^{-2}} = \left(-t\sqrt{1+t^2}, 0, \sqrt{1+t^2}\right)$

2) $A = lw$ so $\dfrac{dA}{dt} = \dfrac{\partial A}{\partial l}\dfrac{dl}{dt} + \dfrac{\partial A}{\partial w}\dfrac{dw}{dt}$

$= w\dfrac{dl}{dt} + l\dfrac{dw}{dt} = 8(3) + 10(-2) = 4\ \text{cm}^2/\text{sec}$

increasing since positive.

3) $x_{CM} = \dfrac{\iint x^2\,dA}{\iint x\,dA}$ $\quad y_{CM} = \dfrac{\iint xy\,dA}{\iint x\,dA}$

$\iint x^2\,dA = \int_0^2\int_{x^2}^{x+2} x^2\,dy\,dx = \frac{44}{15}$

$\iint x\,dA = \int_0^2\int_{x^2}^{x+2} x\,dy\,dx = \frac{8}{3}$

Integral evaluated on Mathematica.

$\iint xy\,dA = \int_0^2\int_{x^2}^{x+2} xy\,dy\,dx = 6$

so $x_{CM} = 11/10$ $\quad y_{CM} = \frac{9}{4}$

Rev Answer cont

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4a) $\nabla w = (y+z, x+z, y+x)\Big|_{(1,2,-1)} = (1, 0, 3)$

$u = \frac{(1, 2, -2)}{3}$ so $D_u = (1, 0, 3)\cdot\left(\frac{1}{3}, \frac{2}{3}, \frac{-2}{3}\right) = \frac{-5}{3}$

b) Increases most quickly in direction of gradient, decreases most quickly in direction of $-\nabla w = (-1, 0, -3)$ - can write this as unit vector - direction of $\left(\frac{-1}{\sqrt{10}}, 0, \frac{-3}{\sqrt{10}}\right)$

Magnitude of decrease = magnitude of gradient = $\sqrt{10}$

c) Gradient is normal to level surface, so normal is $(1, 0, 3)$

Equation of plane is $(1, 0, 3)\cdot(x-1, y-2, z+1) = 0$

or $x-1+3(z+1) = 0$ or $x+3z+2=0$

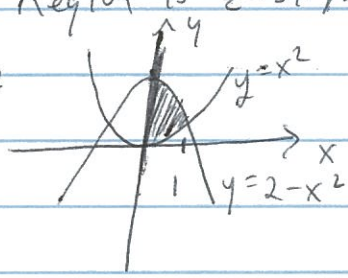
5) Region is z-simple with $0 \le z \le 1-x$ over region

$\int_0^1\int_{x^2}^{2-x^2}\int_0^{1-x} 1\, dz\, dy\, dx = \frac{5}{6}$ by Mathematica

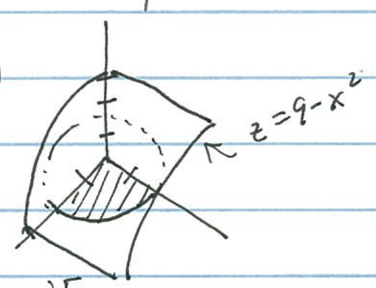
$x^2 = 2-x^2$

$2x^2 = 2$

$x = \pm 1$



6)



Region is z-simple with $0 \le z \le 9-x^2$ over region, 1st quadrant of circle $x^2+y^2=4$

$\int_0^{\frac{\pi}{2}}\int_0^2\int_0^{9-r^2\cos^2\theta} \sin\theta\, r\, dz\, dr\, d\theta = \frac{50}{3}$ by Mathematica

7) a) $\frac{\partial F_2}{\partial x} = 2x \neq 2y = \frac{\partial F_1}{\partial y}$ so since 2-d curl $\neq 0$, not conservative

b) To calculate $\int_C \vec{F}\cdot d\vec{r}$ directly, write $\vec{r}(t) = (2\cos\theta, 2\sin\theta)$

so $\int_0^{2\pi} \vec{F}(r(\theta))\cdot r'(\theta)\, d\theta = \int_0^{2\pi} (4\sin^2\theta, 4\cos^2\theta)\cdot(-2\sin\theta, 2\cos\theta)\, d\theta$

$\int_0^{2\pi} -8\sin^3\theta + 8\cos^3\theta\, d\theta = 0$ (by Mathematica - you do not need to know this integral)

By Green's Theorem $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 2x-2y\, dy\, dx = 0$

Rev Answers ③

8) See answer in text

9) I will write plane in the form $ax+by+z=d$ (to make it cleaner when solve for z). Find volume in the 1st octant.

$$\text{Vol} = \int_0^{d/a}\int_0^{\frac{d-ax}{b}}\int_0^{d-ax-by} 1 \, dz\,dy\,dx = \frac{d^2}{6ab} \text{ by Mathematica.}$$

Since the plane passes through $(1,2,3)$ $d = 3+a+2b$

so $\text{Vol} = \frac{(3+a+2b)^2}{6ab}$. Minimize as a function of a and b

Found critical point on Mathematica $= (3, 1.5)$

There discriminant is 12 and $f_{aa} = 2$ so Minimum.

the plane is $3x + 1.5b + z = 12$

10) a) $2\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{5/6}}$ converges conditionally by alternating series test

Does not converge absolutely since $p = 5/6 < 1$

b) Ratio test $\frac{(n+1)!}{1\cdot 3\cdots(2n-1)(2n+1)} \cdot \frac{1\cdot 3\cdots(2n-1)}{n!} = \frac{n+1}{2n+1} \to \frac{1}{2} < 1$ so converges

c) $\frac{1}{n^3+n} < \frac{1}{n^3}$ converges $p=3$ so original series converges by comparison

11 a)

$f(x) = \ln(2x+1)$	$f(0) = 0$	$a_0 = 0$
$f'(x) = \frac{2}{2x+1}$	$f'(0) = 2$	$a_1 = 2$
$f''(x) = -2^2(2x+1)^{-2}$	$f''(0) = -2^2$	$a_2 = -2^2/2!$
$f'''(x) = 2^3(-2)(2x+1)^{-3}$	$f'''(0) = 2^3 \cdot 2$	$a_3 = 2^3/3$
$f^{iv}(x) = -2^4(-3)(-2)(2x+1)^{-4}$	$f^{iv}(0) = -2^4 3!$	$a_4 = -2^4/4$

$$\ln(2x+1) = 2 + \sum_{n=2}^{\infty} \frac{(-1)^{n-1} 2^n}{n} x^n$$

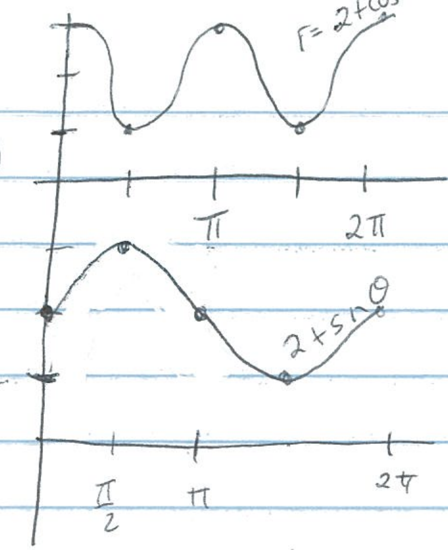
Radius of convergence: $\lim_{n\to\infty} \frac{2^{n+1}|x|^{n+1}}{n+1} \cdot \frac{n}{2^n |x|^n} = 2|x| < 1$ if $|x| < \frac{1}{2}$ converges absolutely

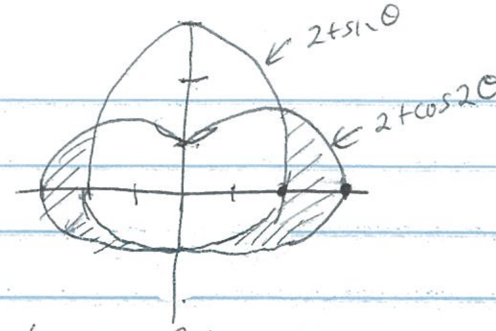
Check endpoints $x = \frac{1}{2}$: $\sum \frac{(-1)^n}{n}$ converges, $x = \frac{-1}{2}$: $\sum \frac{1}{n}$ diverges

so converges on $(\frac{-1}{2}, \frac{1}{2}]$

④

12a)





intersect $2+\cos 2\theta = 2+\sin\theta$

$\cos 2\theta = \sin\theta$

$1-2\sin^2\theta - \sin\theta = 0$

$\sin\theta = \frac{1}{2}$ $\theta = \frac{\pi}{6}$

$\theta = -\frac{\pi}{2}$

b) $2\int_{-\frac{\pi}{2}}^{\pi/6} \frac{1}{2}(2+\cos 2\theta)^2 - \frac{1}{2}(2+\sin\theta)^2 \, d\theta$

↓ Mathematica

$= \frac{51\sqrt{3}}{16}$

13) $G(\theta, r) = (r\cos\theta, r\sin\theta, 3r)$

$T_\theta = (-r\sin\theta, r\cos\theta, 0)$

$T_r = (\cos\theta, \sin\theta, 3)$

$N = (3r\cos\theta, 3r\sin\theta, -r)$

$SA = \int_{1/3}^{2/3}\int_0^{2\pi} \sqrt{10}\, r \, d\theta\, dr = \cancel{2}\pi\sqrt{10} \, \frac{r^2}{\cancel{2}}\Big]_{1/3}^{2/3} = \frac{\pi\sqrt{10}}{3}$

14) a) $r(t) = (3,0,0) + t\left(-3, \frac{\pi}{2}, 3\right) = \left(3-3t, \frac{\pi}{2}t, 3t\right)$ $0 \le t \le 1$

$r'(t) = \left(-3, \frac{\pi}{2}, 3\right)$

$\int_0^1 \left(3t, 3-3t, \frac{\pi}{2}t\right) \cdot \left(-3, \frac{\pi}{2}, 3\right) dt = \int_0^1 -9t + \frac{3\pi}{2} = \frac{-9t^2}{2} + \frac{3\pi}{2}t\Big]_0^1$

$\boxed{\frac{3\pi - 9}{2}}$

Note: 3 missing in problem

b) $r(t) = (3\cos t, t, 3\sin t)$ $0 \le t \le \frac{\pi}{2}$

$r'(t) = (-3\sin t, 1, 3\cos t)$

$\int_0^{\pi/2} (3\sin t, 3\cos t, t) \cdot (-3\sin t, 1, 3\cos t)\, dt$

$\int_0^{\pi/2} -9\sin^2 t + 3\cos t + 3t\cos t \, dt = \frac{-3\pi}{4}$

15) Check curl = 0

$F(x,y,z) = x^2y^3 + xz^2 + zy^2$