Math 204–Block 8, 2016

FINAL REVIEW

The exam will be available by 8AM Wednesday, and due at noon. You will be allowed to use the one page of notes you prepared for yourself in advance. You will not be allowed to use any other books or notes, nor any help from other people. No computers may be used. Calculators may be used for arithmetic and to evaluate trig, exponential and log functions only–no symbolic algebra, graphing, calculus or programmable features. I will give you any unusual integration formulas. You must take the exam in TSC or the fishbowl.

Sections covered: 10.1-10.7; 11.1, 11.2, **11.3,11.4**; 12.1-12.5, **12.7**; 13.1-13.5; 14.3-14.7; **15.1-15.5; 16.1-16.5, 17.1** The sections since the midterm (in bold) will be emphasized.

Practice Problems:

1. $\vec{r}(t) = \langle \frac{t^2}{2}, 0, \frac{t^3}{3} \rangle$ represents the path of an object moving in space where t is time.

a. Find the velocity at time t.

b. Find the speed at time t.

c. Find the distance travelled between $t = 0$ and $t = \sqrt{3}$.

d. Find the curvature of the path.

e. Find the unit normal $\vec{N}$.

2. A rectangular sheet of rubber is being stretched in such a way that when $t = 10$ sec. the length is increasing at the rate of 3cm/sec and the width is decreasing at 2 cm/sec. At this time, the length is 10 cm and the width is 8 cm. Is the area increasing or decreasing at $t = 10$ sec? At what rate?

3. Find the center of mass of the region in the first quadrant bounded by $y = x^2$ and $y = x + 2$ if the density is x.

4. a. Find the rate of change of $w = xy + yz + xz$ at the point (1,2,-1) in the direction of $< 1, 2, -2 >$.

b. Find the direction in which this function DECREASES most rapidly. What is the magnitude of the rate of decrease in that direction?

c. Find the equation of the plane tangent to the level surface of this function, $w = -1$, at (1,2,-1).

5. Use triple integration to find the volume bounded by $x = 0$, $z = 0$, $y = x^2$, $y = 2 - x^2$, and $x + z = 1$.

6. Find the mass of the volume bounded by $x^2 + y^2 = 4$, $z = 9 - x^2$, $y = 0$, $x = 0$, $z = 0$, where density (in cylindrical coordinates) is equal to $\sin\theta$.

7. a. Is the vector field $\vec{F}(x, y) = \langle y^2, x^2 \rangle$ conservative? Prove your answer.

b. Find $\int_C \vec{F} \cdot d\vec{r}$ for the vector field in part a., where C is a circle of radius 2 around the origin, traced in a ccw direction. both by calculating the line integral directly and by using Green's Theorem.

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9. Find an equation of the plane that passes through the point (1,2,3) and cuts off the smallest volume in the first octant.

10. Determine whether each of the following series converges or diverges and give a convincing reason why.

a. $2 - \frac{2}{\sqrt[6]{2^5}} + \frac{2}{\sqrt[6]{3^5}} - \frac{2}{\sqrt[6]{4^5}} \cdots$

b. $\sum_{n=1}^{\infty} \quad \frac{n!}{1 \cdot 3 \cdot 5 \cdots (2n-1)}$

c. $\sum_{n=1}^{\infty} \quad \frac{1}{n^3+n}$

11. a. Find the Taylor series expansion for the function $f(x) = ln(2x+1)$ centered at 0.

b. Find for which x this series converges.

12. a. Graph the two polar curves $r = 2 + \cos 2\theta$ and $r = 2 + \sin\theta$ without the use of your calculator.

b. Find the area inside the first graph but outside the second.

13. Find the surface area of the part of the cone $z^2 = 9(x^2 + y^2)$ that lies between the planes $z = 1$ and $z = 2$.

14. Find the work done by the force field $\vec{F}(x, y, z) = \langle z, x, y \rangle$ from the point (3,0,0) to the point $(0, \pi/2, 3)$ along

a. a straight line

b. the helix $x = 3\cos t$, $y = t$, $z = \sin t$.

15. Show that $\vec{F}(x, y, z) = \langle 2xy^3 + z^2, 3x^2y^2 + 2yz, y^2 + 2xz \rangle$ is conservative, and find a function f such that $\vec{F} = \nabla f$.