

Alexander Horned Sphere

This sphere separates S^3 into exactly two components, an interior and an exterior. The closure of the interior (bounded) is identical to a cube. The closure of the exterior (contain the point at infinity) is not. Call the closure of the interior C^3 .

We can decompose S^3 into mutually disjoint sets where one set is C and all the rest are singletons. In other words, we identify the closure of interior of the Alexander horned sphere to a single point in the quotient space.

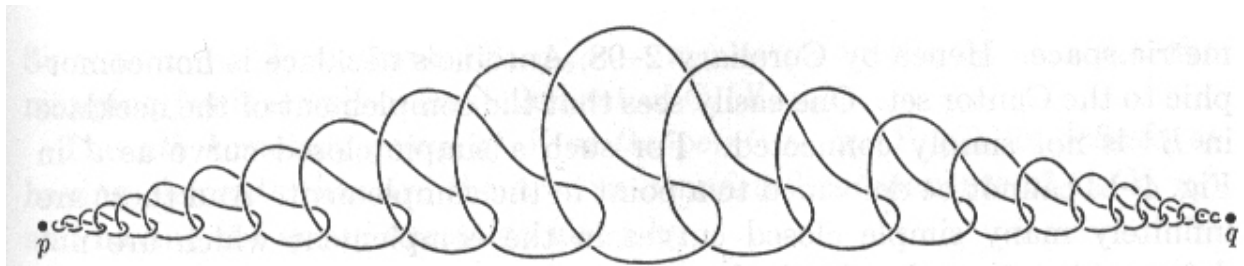
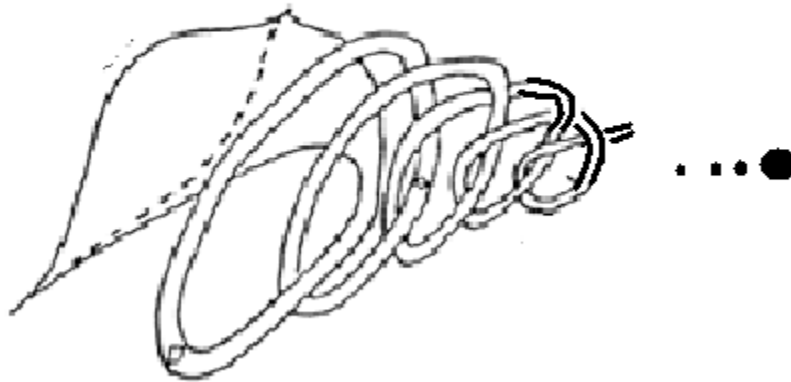


FIG. 4-12. A simple arc in E^3 whose complement is not simply connected.

This is a wild arc discovered about 70 years ago by Fox and Artin. It can be altered to become another example of a wild sphere in S^3 .

Drawing by Duane Loveland



The Fox-Artin Sphere

This sphere is modeled on the right half of the Fox-Artin wild arc above. Its complement is simply connected and, in fact, looks just like Euclidean 3-space!