Separation Properties

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Although point-set topology may be a disease (Poincare), it is interesting, entertaining, and occupies an important place in mathematics. In fact, it marks the intersection between topology and logic (set theory).

In the reading you have learned about a particular type of property that is extremely important in topology. A *Hausdorff* topologoical space $(X, \mathcal{T})$ allows for separation of a pair of points by disjoint open sets. You will explore other separation properties.

Definition 1. *A topological space $(X, \mathcal{T})$ is T_1 if $\{x\}$ is a closed set for each $x \in X$.*

Example 1. *Find an example of a topological space that is not T_1.*

Topologists label Hausdorff spaces as T_2 spaces

Definition 2. *A topological space $(X, \mathcal{T})$ is regular if for each point $x \in X$ and closed subset $E \subset X$ with $x \notin E$, there exist open sets U_1, U_2 in X such that $x \in U_1$, $E \subset U_2$, and $U_1 \bigcap U_2 = \emptyset$.*

In other words in a regular space points and closed sets can be separated by open sets.

Definition 3. *A topological space $(X, \mathcal{T})$ is T_3 if it is both regular and a T_1 space.*

Example 2. *Find an example of a regular space that is not T_1.*

Definition 4. *A topological space $(X, \mathcal{T})$ is normal if for each pair of non-intersecting closed sets E_1, E_2, there exist open sets U_1, U_2 with $E_1 \subset U_1$, $E_2 \subset U_2$, and $U_1 \bigcap U_2 = \emptyset$*

Definition 5. *A topological space $(X, \mathcal{T})$ is T_4 if it is normal and T_1.*

Example 3. *Find an example of a normal space that is not T_4.*

Example 4. *Find an example of a T_2 space that is not T_3.*

Example 5. *Find an example of a T_3 space that is not T_4.*