Math 400 Topology Homework Week 3 Block 6 2009-10

Homework:

Instructions: The same rules apply.

R 3
Reading: 7.1; 7.2(thru Theorem 7.13)
Exercises: 7.1&, 7.2(a)&, 7.2(b)*, 7.3&, 7.7&, 7.8*

F 3
Reading: Finish 7.2; 7.5
Exercises: 7.11, 7.12*, 7.15, 7.17, 7.21*, 7.37*, 7.40*, *Extra 2*

Extra 2: Let γ be a limit ordinal on the Long Line (LL). Prove $\{x \in LL \mid 0 \leq x \leq \gamma\}$ is a compact subset of the long line.

F&F: Fame and fortune awaits the solvers of these problems.

F&F.1: Do the two sets $\mathbb{R}$ and $\mathbb{R}\times\mathbb{R}$ have the same cardinality? If your answer is yes, then you must exhibit a bijection between the two. If your answer is no, then you must show no bijection exists. You may collaborate with one other person on this one if you so choose. **It is due by Friday of Week 2 (F2)**

F&F 2: Let (X,d) be a metric space And S b e the set of Cauchy sequences in X. Define a relation ~ on S by declaring $\{s_k\}\sim\{t_k\}$ to mean that $d(s_k,t_k) \to 0$ as $k\to \infty$. **This is due at 9:30 am on R3**.

(a) Prove that the relation ~ is an equivalence relation.
(b) Let X* denote the set of equivalence classes of S and let s* denote the equivalence class of s = $\{s_k\}$. Show that the function $\rho(s^*,t^*) = \lim_{k\to\infty} d(s_k,t_k)$ (for $s^*,t^* \in X^*$ defines a metric on X*.
(c) Show that (X^*,ρ) is complete.
(d) For $x\in X$ define x* to be the constant sequence {x,x,x, …}. Prove that for $x,y\in X$, $d(x,y) = \rho(x^*,y^*)$.

F&F 3: Prove that Tuhp is T3 but not T4. **This is due at 9:30 AM on F3.**

F&F 4: Suppose X is a compact, connected metric space with exactly two non-cut points. Prove that X is homeomorphic to the unit interval [0,1].