Math 400 Topology Homework Week 2 Block 6 2009-1	1ath 400	Topology	Homework	Week 2	Block 6	2009-10
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### Homework:

Instructions: The same rules apply The reading is due at the beginning of class on the given day. However, there are three types of exercises. I want oral presentations in a problem session on or after the given day of those marked with a pound sign (#); I want written solutions on the given day to those that are underlined by 3:30 PM; I expect class discussion on the given day about those marked with an ampersand (&), and I want written solutions to those marked with an asterisk (\*) by 9:30 AM on the following Monday. You may neither collaborate with other persons nor consult sources outside the course materials on the problems marked with an asterisk. I encourage you to discuss the others with your classmates or with me. However, your presentation or write-up should be your own.

There will be several F&F problems assigned during the block. I would like you to work on at least two of these (of your own choosing). I will specify the application of the Honor Code and the due date for each one.

### M 2

Reading: Finish 3.1; finish 3.2; 3.3 Exercises: <u>1.27</u>, <u>2.28(efg)</u>, <u>3.3</u>, <u>3.14</u>

# T 2

Reading: 3.4 (we will divide up the examples to present); 4.1-4.2 Exercises: 3.1&, 3.2&, 3.5#, 3.7#, 3.8#, 3.10#, 3.12#, 3.15&, 3.18#, 3.23&, 3.24#

### W2 10:00 AM: PROJECT AND EXAMPLE PRESENTATIONS

Reading: None due Exercises: None due

# R2

Reading: 5.3 - 5.4 (most of this is review); 6.1 – 6.2 Exercises: 3.17#, 3.20#, 3.27#, 3.30#, 3.33&, 3.35#, 3.37\*, 4.2&, 4.3\*, 4.6(a)#, 4.6(b)\*, 4.6&, 4.13(a)#, 4.29#

# F2

Reading: Read p.153-54; reread p. 177-78; 6.3, 6.4(thru Theorem 6.28) Exercises: <u>4.25</u>, <u>4.36</u>, <u>5.20</u>, 5.21&, 6.1&, <u>6.2</u>

### M3

Reading: Finish 6.4 Exercises: (due at 1:00 PM) 6.7#, 6.8#, 6.9#, 6.20&, 6.24#, 6.39#, 6.40#

### **T3**

Midterm exam

### Additional problems:

Extra 1: Find the examples requested on the handout about separation properties (link) in topological spaces.

### F&F: Fame and fortune awaits the solvers of these problems.

F&F.1: Do the two sets  $\mathbb{R}$  and  $\mathbb{R} \times \mathbb{R}$  have the same cardinality? If your answer is yes, then you must exhibit a bijection between the two. If your answer is no, then you must show no bijection exists. You may collaborate with one other person on this one if you so choose. It is due by Friday of Week 2 (F2)

F&F 2: Let (X,d) be a metric space And S b e the set of Cauchy sequences in X. Define a relation ~ on S by declaring  $\{s_k\} \sim \{t_k\}$  to mean that  $d(s_k,t_k) \rightarrow 0$  as  $k \rightarrow \infty$ . This is due at 9:30 am on R3.

- (a) Prove that the relation  $\sim$  is an equivalence relation.
- (b) Let X\* denote the set of equivalence classes of S and let s\* denote the equivalence class of s =  $\{s_k\}$ . Show that the function  $\rho(s^*,t^*) = \lim_{k\to\infty} d(s_k,t_k)$  (for s\*,t\*  $\in$  X\* defines a metric on X\*.
- (c) Show that  $(X^*,\rho)$  is complete.
- (d) For  $x \in X$  define  $x^*$  to be the constant sequence  $\{x, x, x, ...\}$ . Prove that for  $x, y \in X$ ,  $d(x,y) = \rho(x^*, y^*)$ .

F&F 3: Prove that Tuhp is T3 but not T4. This is due at 9:30 AM on F3.