Math 375 Analysis Set Equivalences 2008-09 Tinsley

This is a summary of basic definitions and concepts associated with the notion of equivalence of sets. It includes Class Lemma 5 (along with its proof) that you may find handy for Extra Problem 12.

Definition 1. *Sets S and T are equivalent if there exists a one to one, onto function $\varphi : S \twoheadrightarrow T$.*

This is obviously a symmetric relation. If the S and T are equivalent, then T and S are equivalent.

Definition 2. *A set S is finite if it is equivalent to the subset $\{1, 2, 3, \cdots, n\}$ of $\mathbb{Z}$ for some $n \in \mathbb{Z}$.*

Definition 3. *A set S is infinite if it is not finite.*

Definition 4. *A set S is countably infinite if it is equivalent to the set of positive integers $\mathbb{Z}^+$.*

Definition 5. *A set S is countable if it is either finite or countably infinite.*

Class Lemma 5: If S is an infinite set, then S has a subset A that is countably infinite.

Proof. S is non-empty. Let $a_1 \in S$ and $A_1 = \{a_1\}$. Suppose $A_k \subset S$ has been constructed. Since S is infinite, then $S - A_k \neq \emptyset$. Choose $a_{k+1} \in S - A_k$, and let $A_{k+1} = \{a_{k+1}\} \bigcup A_k$. This process can never terminate. Finally, let $A = \bigcup_{k=1}^{\infty} A_k$ $\square$