Math 375 Analysis Final Exam Review 2008-09 Tinsley

The exam will be closed-book, closed-notes, and closed-classmate with the exception of one 3"x5" card on which anything may be written. The questions will be similar in nature to those from the midterm. I will expect you to know the basic definitions and to be able to construct proofs from those definitions. I will not expect you to cite the various theorems that we have proved.

The exam will focus on chapters 6 and 7 from the text. Chapter 5 (Taylor's Theorem) will not be included. Here is a list of terms whose definitions I will expect you to know:

Sequence, subsequence, bounded sequence, convergent sequence, Cauchy sequence, continuous function, interval, metric space, metric, equivalent metric, complete metric space, open set, closed set, bounded set, connected subset, compact subset, equivalent sets, finite set, infinite set, countably infinite set, countable set

You should be familiar with the following examples:

$(\mathbb{R}^n, |x-y|)$, $(\mathbb{R}^n, taxicab)$, $(\mathbb{R}^n, sup)$, $(\mathbb{R}_d, discrete)$

Here are some problems that I considered including on the exam:

1. Give the definition of a compact subset of a metric space. Prove directly from this definition (do not cite theorems) that a finite subset of a metric space is compact.
2. Suppose a sequence $\{s_k\}$ converges to s in metric space, X. Let $A = (\bigcup_{k=1}^{\infty} \{s_k\}) \bigcup \{s\}$. Prove that A is a closed subset of X.
3. Prove that for any sequence of real numbers $\{s_n\}$, $\limsup s_n \geq \liminf s_n$.