1. a) Given $\epsilon > 0$ $\exists$ a $\delta > 0 \ni |f(x) - f(a)| < \epsilon$

1/2 whenever $|x-a| < \delta$

b) $|x^4 - a^4| = |x-a||x^3 + x^2a + xa^2 + a^3|$

1 Let $\delta_0 = 1$; $|x-a| < 1 \Rightarrow |x| < \max(|a+1|, |a-1|) = N$

$|x-a||x^3 + x^2a + xa^2 + a^2| \leq |x-a| \cdot 4 \cdot N^3$

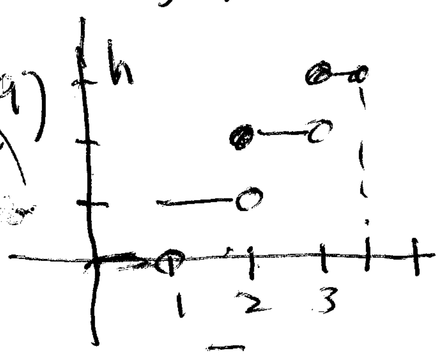
Choose $\delta = \dfrac{\epsilon}{4N^3}$

$|x-a| < \delta \Rightarrow |x-a| < \dfrac{\epsilon}{4N3} = |x-a| \cdot 4N^3 < \epsilon$

$\Rightarrow |x^4 - a^4| < \epsilon$

1

2. a) 1/4



b) Let $\epsilon > 0$

let $\delta = \min\{\epsilon/3, \frac{1}{4}\}$

$P = \{2, 3-\delta, 3+\delta, 3.5\}$

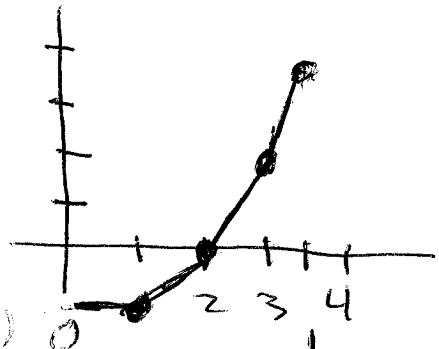
$\overline{S}(p) = 2(3-\delta) + 3 \cdot (2\delta) + 3 \cdot (3.5 - 3 + \delta)$

Subtract $\underline{S}(p) = 2(3-\delta) + 2 \cdot (2\delta) + 3 \cdot (3.5 - 3 + \delta)$

$\overline{S}(p) - \underline{S}(p) = 0 + 2\delta + 0 \leq \frac{2\epsilon}{3} < \epsilon$

1/2 c) $H(x) = \int_2^x h = \begin{cases} 2(x-2) & 2 \leq x < 3 \\ 3(x-3)+2 & 3 \leq x \leq 3.5 \\ (x-2) & 1 \leq x < 2 \\ -1 & 0 \leq x < 1 \end{cases}$

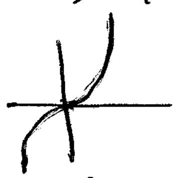
d.



1/4 $H'(x) = \begin{cases} 0 & 0 < x < 1 \\ 1 & 1 < x < 2 \\ 2 & 2 < x < 3 \\ 3 & 0 < x < 3.5 \end{cases}$

undef. at 0, 1, 2, 3

3. a.



1/4

b) $\lim_{x \to 0^+} \frac{x^2}{x} = 0$ 3/4

$\lim_{x \to 0^-} \frac{-x^2}{x} = 0$

So, $\lim_{x \to 0} \frac{g(x) - g(0)}{x - 0} = 0$

c) $g'(x) = 2x$ for $x > 0$

$g'(x) = 0$ for $x = 0$

1/2 $g'(x) = -2x$ for $x < 0$

$g''(0)$ is undefined; the left & right limits are not equal