Math 375 Analysis Homework Week 3 2008-09 Tinsley

Instructions: The same rules apply with one additional designation for exercises. A double asterisk ** means the problem is for extra credit. The work must be entirely your own with only your instructor, class notes, and/or class text for reference.

M3
Exercises: 2.4.3*, Example 4* are due at 9:00 AM.

T3
Reading: 5.1, 6.1-6.2, 7.3, 7.4 (just enough to do the two exercises below)
Exercises: 3.3.1# (p. 42), 7.3.1#, 7.4.1#, Extra 10#, Extra 11#

Extra 10: Let $f(x) = \begin{cases} \sin\frac{1}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$. Prove $\int_0^1 f$ exists.

Extra 11: Read section 3.5. Do exercises 3.5.2 and 3.5.4.

W3
Reading: 6.3, finish 7.3
Exercises: 6.1.1 ((2) and (4) only), 6.2.1, 6.2.2, 6.2.4**, 6.2.5*

R3
Reading: 6.4 (pp. 98-100), 7.4
Exercises: Extra 12*, Extra 13#, 6.3.1#, 6.3.2#, 6.3.4#, 6.3.5#

Definition: A set S is countable if it is either countably infinite or finite.

Extra 12: Prove that a set S is infinite if and only if it is equivalent to a proper subset of itself.

Extra 13: Prove that if S and T are both countable, then $S \cup T$ is countable.

F3:
Reading: 6.4 (p. 101 – 103), 7.5 (p. 132 - 33); 7.6 (p. 134 - 35)
Exercises: Example 6.4.1 (verify the claims), Example 6.4.2&, Exercise 6.4.3&, 6.4.4, 6.4.9*, Extra 14*

Extra 14: We discussed in class three metrics for the plane: the straight line metric, the taxi cab metric, and the sup metric. Prove that each pair of these metrics is equivalent.

Some interesting questions:

1. Suppose f is continuous function on *(0,1)*. Must f be differentiable at some point in *(0,1)*?

2. Let $S = \{\sin(n) \mid n \text{ an integer}\}$. Find $\sup S$.

3. A subset E of $\bullet^n$ as a metric space is called *compact* if E is closed and bounded. Let $\{U_\alpha \mid \alpha \in A\}$ be an open cover of E. Then there exists a $\delta > 0$ such that if A is any subset of E of diameter less than or

equal to δ , then $A \subset U_\alpha$ for some $\alpha \in E$. (Hint: If you want to start working on this one, then assume *n=1* so that you are in the real numbers. You already know what a bounded set is. A subset of the real numbers is open if it is the union of open intervals. A subset of real numbers is closed if its complement is open.)