MA 375 Modern Analysis Homework Week 2 Tinsley

Instructions: The same rules apply.

M 2
Reading: review section 3.2 (study Theorem 3.2.2); section 4.1
Exercises: Example 3, 3.1.1* (Hint: use Axiom 3.1.1), 3.1.2#, 3.1.3#, 3.1.5&, 3.2.1#
Note: The problems just above marked with a # are due at the problem session on T 2.

Example 3: Let $h(x) = \begin{cases} x^2 \cdot \sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$. Use the ε-δ definition (Definition 1.2.1) of limits to calculate $h'(0)$. (Hint: don't try to use any rules.)

T2
Reading: sections 4.2, 4.6
Exercises: 2.4.3*, ~~3.2.3#~~, Extra 7#, ~~4.2.1#~~

Extra 7: Let h(x) be the function from Example 3. You showed that $h'(0)$ exists and calculated its value. Now, calculate $h'(x)$ for all real numbers x (you may use rules). Be sure to explain how you arrived at your answer. Prove that $h'(x)$ is not continuous at $x = 0$. In your proof, you may cite previous work.

W 2
Reading: sections 5.1, 7.4
Exercises: 2.4.4, 3.2.2, Example 4*

Example 4: Define $f : [0,1] \to \bullet$ by $f(x) = \begin{cases} 0 & \text{if x is rational} \\ 1 & \text{if x is irrational} \end{cases}$. Make a conjecture about whether *f* is Riemann integrable. Prove your conjecture.

R 2
Reading: no new
Exercises: 4.2.2#, 4.2.4#, 4.2.5#, Example 5#, Extra 8#, Extra 9a#, Extra 9b#

Example 5: Let $g(x) = \begin{cases} 0 & \text{for } x \in \left\{1 - \frac{1}{n} \mid n = 2,3,\cdots\right\} \\ x & \text{elsewhere} \end{cases}$. Calculate $\int_0^1 f$.

Extra 8: For the function *g(x)* in Example 5. Let $G(x) = \int_0^x g$. Calculate $G'(x)$.

Extra 9: This replaces exercise 3.2.3.
a. Let $f(x) = 1/x$ for $0 < x \leq 1$. Prove that f is not uniformly continuous.

b. Let g(x) = sin(1/x) for $0 < x \leq 1$. Prove that g is not uniformly continuous even though *g* is bounded.

F2
Midterm exam, Fearless Friday student seminar, math picnic

M3
2.4.3*, Example 4* are due at 9:00 AM. You may consult only me, your text, and/or class notes.

Some interesting questions:

1. Suppose f is continuous function on $(0,1)$. Must f be differentiable at some point in $(0,1)$?

2. Let $S = \{\sin(n) \mid n \text{ an integer}\}$. Find $\sup S$.