

Instructions: The reading is due at the beginning of class on the given day. However, there are three types of exercises. I want oral presentations in a problem session on or after the given day of those marked with a pound sign (#); I want written solutions on the given day to those that are underlined by 3:00 PM; I expect class discussion on the given day about those marked with an ampersand (&), and I want written solutions to those marked with an asterisk (*) by 9:00 AM on the following Monday. You may neither collaborate with other persons nor consult sources outside the course materials on the problems marked with an asterisk. I encourage you to discuss the others with your classmates or with me. However, your presentation or write-up should be your own.

We will discuss specific expectations for each type as we go along.

M 1 (i.e. Monday of Week 1)

Reading: p. 3-5 (up thru Def 2.1); p. 123-24

Exercises: 1.1.1&, 7.1.1&, Example 1&

Example 1: (Analyst's function) $f(x) = \begin{cases} \sin \frac{1}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$

- Sketch the graph of $f(x)$ over the interval: $-\frac{2}{\pi} < x < \frac{2}{\pi}$.
- What is $f(x)$ for $x \in \left\{ \frac{1}{\pi}, \frac{1}{2\pi}, \frac{1}{3\pi}, \dots, \frac{1}{n\pi}, \dots \right\}$? Indicate where the points $\left(\frac{1}{n\pi}, f\left(\frac{1}{n\pi}\right) \right)$ (for $n = 1, 2, \dots$) lie on your graph in part (a).

T 1

Reading: section 1.2 p. 5-6; sections 7.1-7.2 (p. 125-28)

Exercises: 1.1.2#, 1.1.3#, 1.2.1#, 7.1.2#, Example 2#, Extra 1*

Example 2: Let $g(x) = \begin{cases} x \cdot \sin \frac{1}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$; sketch the graph of $g(x)$ over the interval $-\frac{2}{\pi} \leq x \leq \frac{2}{\pi}$.

Extra 1: Conjecture whether $g(x)$ of Example 2 has a derivative at $x = 0$. Prove or disprove your conjecture.

W 1

Reading: sections 1.4 (chapter 1 section 4); 2.1, 2.2 (chapter 2 sections 1 and 2)

Exercises: 7.1.4, 7.1.6, 7.2.1, 7.2.2&, Extra 2

Extra 2: Let $g(x)$ be the function from Example 2. Based on your graph, make a conjecture about whether $\lim_{x \rightarrow 0} g(x)$ exists. If you believe it exists, then give its value. Briefly explain your answer. You need not provide a formal proof (yet, anyway ☺).

R 1

Reading: section 2.3 (p. 22 only); section 2.6; section 7.3 (thru Theorem 3.2 only)

Exercises: 2.1.1#, 2.1.2#, 2.2.1#, 2.2.2#, 2.2.3#, 2.2.4&, Extra 3*

Extra 3: Let $g(x)$ be the function from Example 2. Use the ε - δ definition (Definition 1.2.1) of limits to prove that $\lim_{x \rightarrow 0} g(x) = 0$.

F 1

Reading: section 3.1; section 3.2 (thru Theorem 2.3)

Exercises: Extra 4, Extra 5, Extra 6*

Extra 4: Use the ε - δ definition (Definition 1.2.1) of limits to prove that $\lim_{x \rightarrow a} x^3 = a^3$. Does the choice of δ for a given ε depend on the value of a ? Explain.

Extra 5: Let $f(x)$ be the function from Example 1. Prove that $\lim_{x \rightarrow 0} f(x)$ does not exist. (Hint: we talked in class on Tuesday about a strategy for this proof.)

Extra 6: Use the ε - δ definition (Definition 1.2.1) of limits to prove that $\lim_{x \rightarrow a} \sqrt{x} = \sqrt{a}$. Assume that $a > 0$. Does the choice of δ for a given ε depend on the value of a ? Explain.

M 2

Reading: sections 4.1 and 4.2 (the integral appears! ☺)

Exercises: Example 3, 3.1.1* (Hint: use Axiom 3.1.1), 3.1.2#, 3.1.3#, 3.1.5&, 3.2.1#

Note: The problems just above marked with a # are due at the problem session on T 2.

Example 3: Let $h(x) = \begin{cases} x^2 \cdot \sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$. Use the ε - δ definition (Definition 1.2.1) of limits to calculate $h'(0)$. (Hint: don't try to use any rules.)

Here is a separate list of the problems due Monday at 9:00 AM. The work on these must be entirely your own. You may consult only your text or me: Extra 1, Extra 3, Extra 6, 3.1.1