47 67

Math 126 Calculus I Midterm Exam Block 7 2008-09 Part I

42.5↑
37.5↑
30.5↑
26↑

Write down the time that you begin the exam ____________________

Instructions: This is a closed-book, closed-notes, closed calculator, closed computer, and closed-neighbor exam with the exception of one 3"x5" index card on which anything may be written. Show all relevant work to receive full credit. Include a signed statement of the Honor Code. Part I of the exam is due one hour and thirty minutes after you begin after which you can pick up Part II at my office.

60.5 ↑ 8
53.5 ↑ 9
43.5 ↑ 5
37 ↑ 0

22

1. (5 points)
a. State the limit definition of the **derivative** of a function $y = f(x)$.

1.5 $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ (.5)

b. Let $y = f(x) = \frac{1}{(x-1)^2}$; calculate the derivative of $f(x)$ **directly from the limit definition**. You will receive no credit for calculating the derivative using rules.

$$\frac{f(x+h)-f(x)}{h} \text{ (.5)} = \frac{\frac{1}{(x+h-1)^2} - \frac{1}{(x-1)^2}}{h} \text{ (1.5)} = \frac{(x-1)^2-(x+h-1)^2}{h(x+h-1)^2(x-1)^2} \text{ (1)}$$

$$= \frac{x^2-2x+1-(x-1)^2-2(x-1)h-h^2}{h(\ )(\ )} \text{ (.5)} = \frac{-h(2(x-1)+h)}{h(\ )(\ )} \text{ (1.5)}$$

$$= \frac{-2(x-1)-h}{(\ )(\ )} \to \frac{-2(x-1)}{(x-1)^4}$$

$(x+h-1)$
$(x+h-1)$
$-x+h+1$
$xh + h^2 - h$
$x^2 \ xh \quad -x$

$= x^2+2xh+h^2+1$
$-2x-2h$

2. (20 points) In each case, find $\frac{dy}{dx}$ using any method. You must find an expression for $\frac{dy}{dx}$, but **you need not simplify your answers**. There are two more parts on the next page.

a. $y = x^3 - 4x + \frac{2}{x} - \ln(x) + 4 - \tan^{-1}(3x)$

$y' = 3x^2 - 4 - \frac{2}{x^2} - \frac{1}{x} + 0 - \frac{3}{1+9x^2}$

1 .5 1 1 .5 1

10

b. $y = e^{-x^2} \sin(1-x^3)$

$y' = -2xe^{-x^2}\sin(1-x^3) + e^{-x^2}\cos(1-x^3)(-3x^2)$

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c. $y = \sqrt{1 - x^2}$

$y' = \frac{1}{2}(1-x^2)^{-1/2}(-2x)$

10

d. $y = 4\cos(1 + e^x)$

$y' = -4\sin(1+e^x)e^x$

4. (5 points) The basic rules of differentiation allow us to compute the derivatives for all the other trig functions. Recall that $\cot(x) = \dfrac{\cos(x)}{\sin(x)}$.

a. Use the quotient rule to find $\dfrac{d}{dx}(\cot(x))$.

2.5 1.5

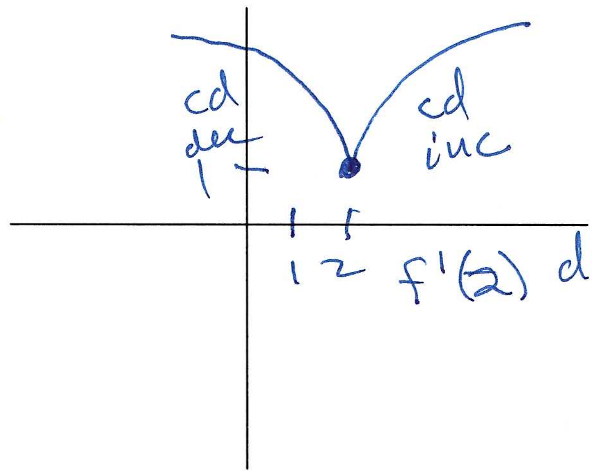
$(\cot(x))' = \dfrac{\sin(x)(-\sin(x)) - \cos(x)\cos(x)}{\sin^2(x)} = \dfrac{-1}{\sin^2 x}$

$= -\csc^2(x)$ (1)

b. Show that your answer to (a) is equal to $-\csc^2(x)$ where $\csc(x) = \dfrac{1}{\sin(x)}$. Do not forget that $\sin^2(x) + \cos^2(x) = 1$.

6. (4 points) On the axes below, sketch the graph of a function, $y = f(x)$, that satisfies all of the following four conditions simultaneously (yes, there are many possibilities).

$f(2) = 1$ is a local minimum
$f''(x) < 0$ for $x \neq 2$
$f'(x) < 0$ for $x < 2$
$f'(x) > 0$ for $x > 2$



f'(2) dne

7. (6 points) Use implicit differentiation to find to find $\frac{dy}{dx}$. You must solve for $\frac{dy}{dx}$, but you need not simplify your solution.

$$xy^3 - \sin(x^2 y) + e = \ln(x) + 3x$$

$$y^3 + x \cdot 3y^2 \cdot y' - \cos(x^2y)\left(2xy + x^2y'\right) + 0 = \frac{1}{x} + 3$$

$$3xy^2y' - x^2\cos(x^2y) \cdot y' = \frac{1}{x} + 3 - y^3 + 2xy\cos(x^2y)$$

$$y' = \frac{\frac{1}{x} + 3 - y^3 + 2xy\cos(x^2y)}{3xy^2 - x^2\cos(x^2y)}$$

8. (3 points) Approximate $\cos\left(\frac{9\pi}{24}\right)$ using the linearization of y = cos(x) at $x = \frac{\pi}{3}$ (hint: thus, $\Delta x = \frac{\pi}{24}$). Your answer need not appear in decimal form. DO NOT USE YOUR CALCULATOR. You may use the back if necessary.

(1)

$$y' = -\sin(x)\Big|_{x=\pi/3} = -\frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3} \quad \Delta x = \frac{\pi}{24} \quad x + \Delta x = \frac{9\pi}{24}$$

$$\cos\left(\frac{9\pi}{24}\right) \doteq y + dy = \cos\left(\frac{\pi}{3}\right) + dy = \frac{1}{2} + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\pi}{24}\right)$$

(1)

$$= \frac{1}{2} - \frac{\sqrt{3}\pi}{48} \quad (1)$$

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Math 126 Calculus I Final Exam Block VII 2008-09 Part II

Record what time you begin the second part __________________

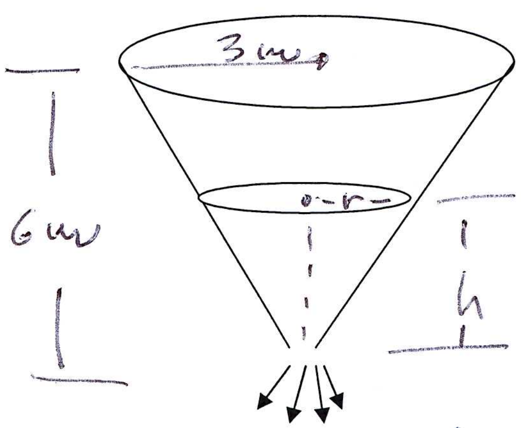
Instructions: This is a closed-book, closed-notes, and closed-neighbor exam with the exception of your 3”×5” index card. You also may use your calculator. Show all relevant work to receive full credit. Include a signed statement of the Honor Code. This part of the exam is due one hour and thirty minutes after you begin.

9. (8 points) Water flows out of an inverted conical basin at the rate of 2 meter^3/min. The basin has radius 3 meters and height 6 meters.

a. At what rate is the depth of the water changing when the depth is 3 meters?

b. At what rate is the depth of the water changing at the instant the depth is equal to 0?

$$V = \frac{1}{3}\pi r^2 h$$



Vars

t - min

water: h = ht or depth, r = radius

(2) Const

$\frac{dv}{dt} = -2$

$R = 3$

$H = 6$

Instant

$h = 3$

$h = 0$

$V = \frac{1}{3}\pi r^2 h$

$\frac{r}{h} = \frac{3}{6} = \frac{1}{2}$ (1)

$2r = h$

$2\frac{dr}{dt} = \frac{dh}{dt}$

$V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 \cdot h$

$= \frac{\pi}{12} h^3$ (1)

$\frac{dv}{dt} = \frac{\pi}{12} 3h^2 \cdot \frac{dh}{dt}$ (1)

$\frac{dh}{dt} = \frac{12}{3\pi h^2} \cdot \frac{dv}{dt}$

$= \frac{4}{\pi h^2} \cdot \frac{dv}{dt}$ (1)

(a) $\frac{dh}{dt} = \frac{4}{\pi \cdot 3^2} \cdot (-2)$ 1

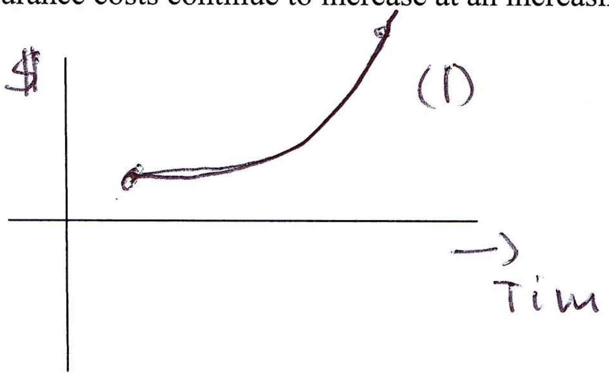
$= \frac{-8}{9\pi}$ m/min $\doteq .28$

(b) $\searrow -\infty$ (1)

8

10. (3 points) Sketch the graph of a function described by the following sentence. Then give a brief description of the graph using terms like increasing, concavity, etc.

"Health insurance costs continue to increase at an increasing rate."

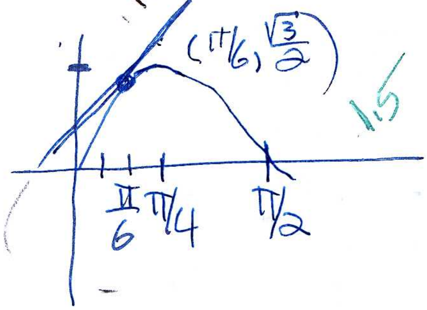


(1)

2 Derivative ↑
⇒ 2nd derivative > 0
⇒ Concave up

11. (4 points) Find the equation of the tangent line to the curve $y = \sin(2x)$ at $x = \frac{\pi}{6}$. You may leave your answer in the point slope form. Plot $y = \sin(2x)$ and the tangent line on the same axes. Label at least

$y' = 2\cos(2x)\Big|_{x=\pi/6} = 2\cos(\frac{\pi}{3}) = 2 \cdot \frac{1}{2} = 1$
(1)



1.5

$y = \sin(2 \cdot \frac{\pi}{6})$
$= \sin(\pi/3) = \frac{\sqrt{3}}{2}$
(.5)

$(y - \frac{\sqrt{3}}{2}) = 1 \cdot (x - \pi/6)$
(1)
4
.523

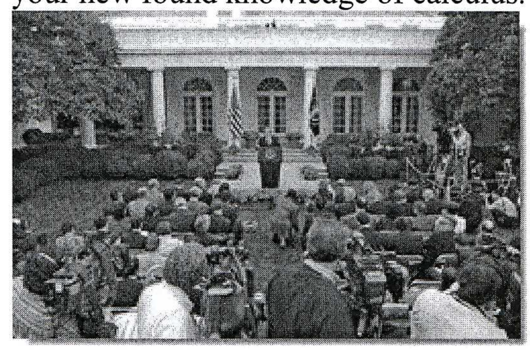
X

12. (5 points)

YEAR	DEBT	D' (Deficit)	D"
1986.75	2125.30		
1987.75	2350.28	238.52	
1988.75	2602.34	253.58	38.49
1989.75	2857.43	315.49	75.18
1990.75	3233.31	403.94	50.08
1991.75	3665.30	415.65	-15.42
1992.75	4064.62	373.09	-50.79
1993.75	4411.49	314.06	-45.92
1994.75	4692.75	281.25	-24.02
1995.75	4973.98	266.03	-30.83
1996.75	5224.81	219.58	-57.67
1997.75	5413.15	150.69	-49.01
1998.75	5526.19	121.56	-38.35
1999.75	5656.27	73.99	-22.98
2000.75	5674.18	75.60	101.52
2001.75	5807.46	277.03	206.14
2002.75	6228.24	487.88	149.19
2003.75	6783.23	575.41	43.43
2004.75	7379.05	574.74	-5.72
2005.75	7932.71	563.96	-18.63
2006.75	8506.97	537.47	97.46
2007.75	9007.65	758.88	
2008.75	10024.72		

The function D = $f(x)$ to the left gives the national debt for each year since 1990. Each of its first two approximate derivatives is also given. The graphs of all three appear on the back of this sheet.

I am asking you to comment on two claims made regarding economic policy in light of your new found knowledge of calculus.



You may use the back of this sheet if necessary.

A. Democrats often claim that the Clinton White House had "balanced the federal budget" by the end of his administration. Moreover, we often hear about the "budget surpluses" under Clinton. Comment on this claim using the above data. Bring to bear your knowledge of calculus. Assume that your audience understands calculus as well.

Deficit never reached a. However it steadily decreased under Clinton, ie, Debt'' < 0.

B. President Bush made the following statement at a news conference early in 2006.

"*I'm going to talk about the pro-growth economic policies that helped bring about the dramatic reduction in the deficit this afternoon, and I'm going to remind our fellow citizens that good tax policy has a lot to do with keeping the economy strong, and therefore, we'll continue to urge the Congress to make the tax cuts permanent.*"

Comment on this statement using the above data. Again, incorporate your knowledge of calculus, assuming that your audience understands calculus as well.

Deficit did decrease slightly but took off again. Inflection point in 2006

C. Compare and contrast the overall performance of these two Presidents regarding the National Debt.

Clinton steadily reduced deficit
Bush: Did decrease it only for a short time.

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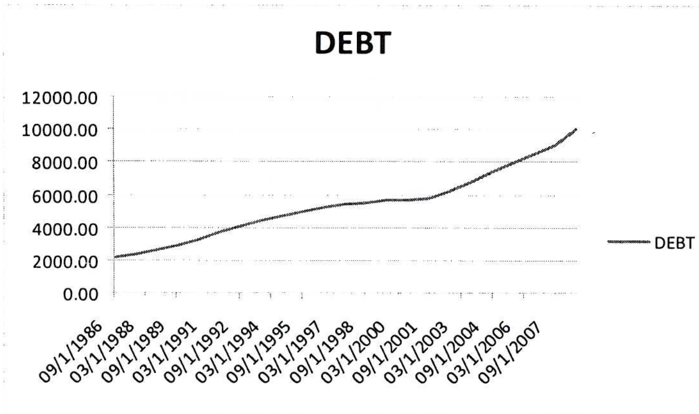
DEBT
12000.00
10000.00
8000.00
6000.00
4000.00
2000.00
0.00
DEBT
09/1/1986
03/1/1988
09/1/1989
03/1/1991
09/1/1992
03/1/1994
09/1/1995
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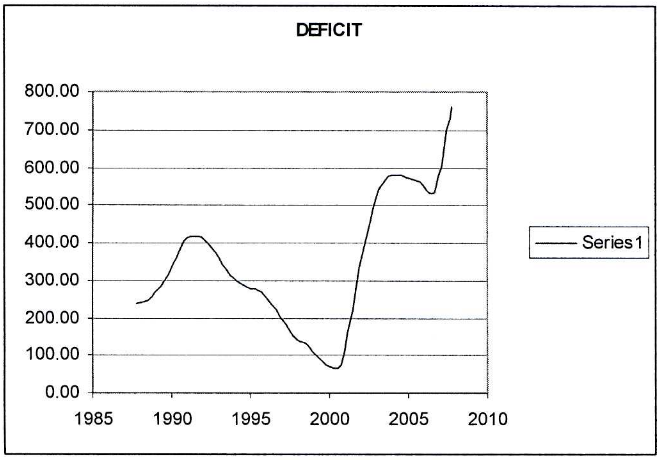
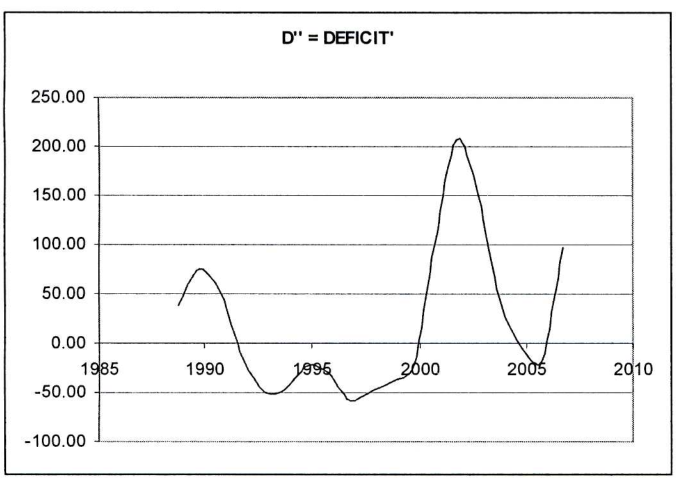
03/1/2006
09/1/2007


DEFICIT
800.00
700.00
600.00
500.00
400.00
300.00
200.00
100.00
0.00
1985
1990
1995
2000
2005
2010
Series1


D'' = DEFICIT'
250.00
200.00
150.00
100.00
50.00
0.00
-50.00
-100.00
1985
1990
1995
2000
2005
2010