

During class you have been applying the ideas of calculus to the AIDS epidemic of the 1980's in the United States. In particular, you took a log transformation and found that growth was not exponential during the early part of the epidemic. In particular, the resulting function was concave down, suggesting that the log transformation was an *over-correction*. In this exercise you will try to find a reasonable *polynomial* model.

Work in groups of two or three (the same groups if possible).

The polynomial model has three parameters (constants) and is of the form $y = f(x) = a(x - c)^b$.

Exercise 1: On your calculator, graph f for several values of a , b , and c until you get a feel for how each affects the function f . You need not turn in this exercise.

The general idea is simple. We know that f' , the derivative of f , has degree one less than f itself. So, we will compute f' numerically and then divide f by f' to obtain a linear function from which we will estimate a , b , and c .

In particular, the derivative of f is $y' = ba(x - c)^{b-1}$. Thus, we see that the ratio $\frac{y}{y'} = \frac{f(x)}{f'(x)}$ is given

by $\frac{a(x - c)^b}{ba(x - c)^{b-1}} = \frac{1}{b}(x - c)$, a linear function of x . This indicates how to find b from the data. You

must *numerically* compute y' , then $\frac{y}{y'}$, and finally $\frac{d}{dx}\left(\frac{y}{y'}\right)$. This last derivative should be equal

to $\frac{1}{b}$ at every points since $\frac{y}{y'}$ is a linear function of x with slope $\frac{1}{b}$. Thus, you can take the mean

or median of these concluding values is an estimate of $\frac{1}{b}$. Finally, take the reciprocal of this

estimate as your estimate of b . This approach has the advantage of using all the data to estimate a single parameter.

Exercise 2: Carry out the process outlined above for the data below. I have subtracted 1980 from each year for ease of computation. DNC stands for "do not compute" (the reason should be obvious from your prior work).

X Year	Y Cases	Y'	Y/Y'	(d/dx)(Y/Y')
0	321	DNC	DNC	DNC
1	1489			DNC
2	4564			
3	10807			
4	22590			
5	41630			
6	70216			
7	105697			
8	148441			DNC
9	197138	DNC	DNC	DNC

What are your estimates of $\frac{1}{b}$ and b ? Briefly reiterate in your own words why the last column on the previous chart contains estimates of $\frac{1}{b}$. You may use a separate sheet or the back of this sheet.

Now, we raise $y = a(x - c)^b$ to the power $\frac{1}{b}$. Then, $y^{\frac{1}{b}} = a^{\frac{1}{b}}(x - c)$. Thus, $y^{\frac{1}{b}}$ is a linear function of x with slope $a^{\frac{1}{b}}$.

Exercise 3: Use the ideas above to find an estimate of a . (Hint: first, transform y ; then differentiate numerically to estimate the slope, $a^{\frac{1}{b}}$. Finally, use your estimates of $\frac{1}{b}$ and $a^{\frac{1}{b}}$ to estimate a .)

X Year	Y Cases	$Y^{(1/b)}$	$(Y^{(1/b)})'$ $a^{(1/b)}$
0	321		DNC
1	1489		
2	4564		
3	10807		
4	22590		
5	41630		
6	70216		
7	105697		
8	148441		
9	197138		DNC

In the final exercise, you are to devise your own method for estimating c , the third parameter.

Exercise 4: Now, you have estimates for a and b . What procedure can you follow to find c ? Explain your choice. Then, use your procedure to find c . Interpret for the layperson the meaning of your value for c in terms of the spread of AIDS.