Cross-Price Elasticity of Demand & Supply and Income Elasticity of Demand

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1. A Brief Review

- **What is elasticity?**

  - **Elasticity is a measure of responsiveness of one variable to another.**

    Example: How much does quantity demanded change when the price of a can of Pepsi goes up by ten cents?

  - **Elasticity measures the proportional (percentage) change in one variable relative to the proportional change in another variable**

- **Why do we use elasticity & not slope to measure responsiveness?**

  Elasticities use percentages that allow us to create a measure of responsiveness that is independent of units.

  Elasticities give us a uniform scale to measure the quantity response of different goods (e.g. apples and beef) to changes in their respective prices.

- **Own-price Demand Elasticity**

  It is the proportional (percentage) change in the quantity demanded of good X divided by the proportional (percentage) change in the price of good X

$$\text{Own Price Elasticity} = \left| \frac{\text{Percentage change in Quantity Demanded}}{\text{Percentage change in Price}} \right|$$

$$E_D = \left| \frac{\Delta Q^D / Q^D}{\Delta P / P} \right| = \left| \frac{\Delta Q^D}{\Delta P} * \frac{P}{Q^D} \right|$$

- **Own-price Supply Elasticity**

  It is the proportional (percentage) change in quantity supplied of good X divided by the proportional (percentage) change in price of good X

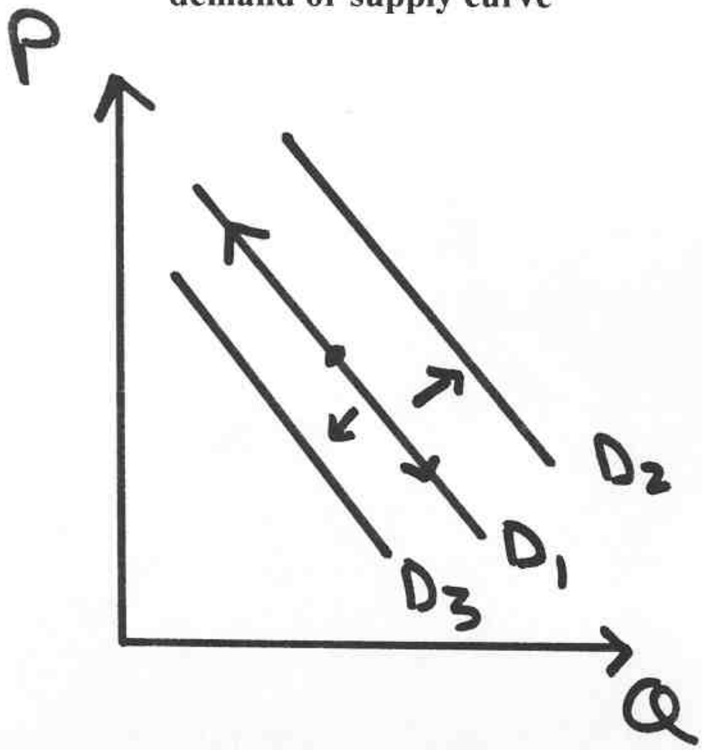
$$\text{Supply Elasticity } = \left| \frac{\text{Percentage change in Quantity Supplied}}{\text{Percentage change in Price}} \right|$$

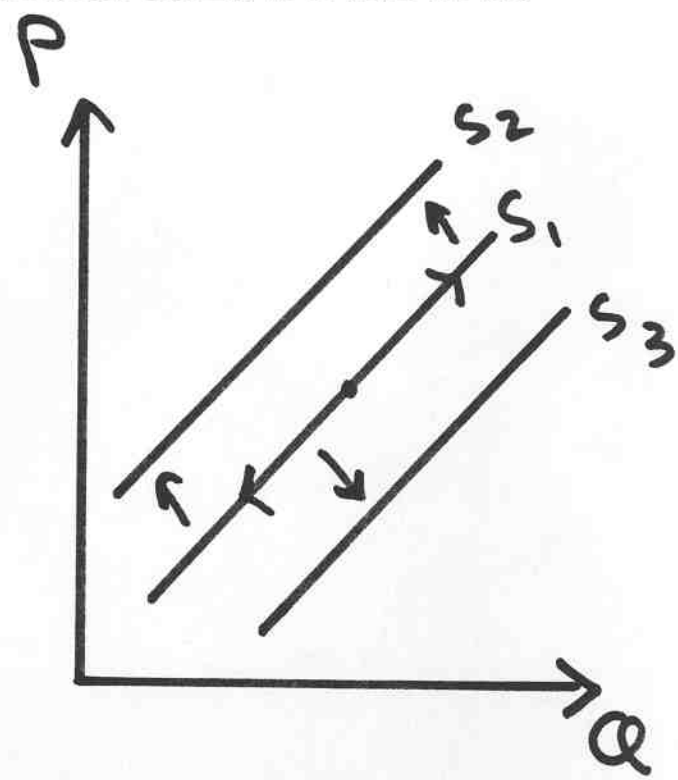
$$E_S = \left| \frac{\Delta Q^S / Q^S}{\Delta P / P} \right| = \left| \frac{\Delta Q^S}{\Delta P} * \frac{P}{Q^S} \right|$$

- **Movements along Vs. Shifts of Demand & Supply Curves**

  - **A change in the price of the good results in a movement along a demand or supply curve. In other words a change in the own-price of the good causes a change in the quantity demanded or quantity supplied.**

  - **A change in any other shifter such as the price of substitutes or complements in consumption or production results in a shift of the demand or supply curve**





2. Cross-price Elasticity of Demand

- **Definition & Formula**

It is the proportional (percentage) change in the demand for good X divided by the proportional (percentage) change in the price of good Y

$$\text{Cross Price Elasticity of Demand} = \frac{\text{Percentage change in the Demand for X}}{\text{Percentage change in Price of Y}}$$

$$E_{XY} = \frac{\Delta X / X}{\Delta P_Y / P_Y} = \frac{\Delta X}{\Delta P_Y} * \frac{P_Y}{X}$$

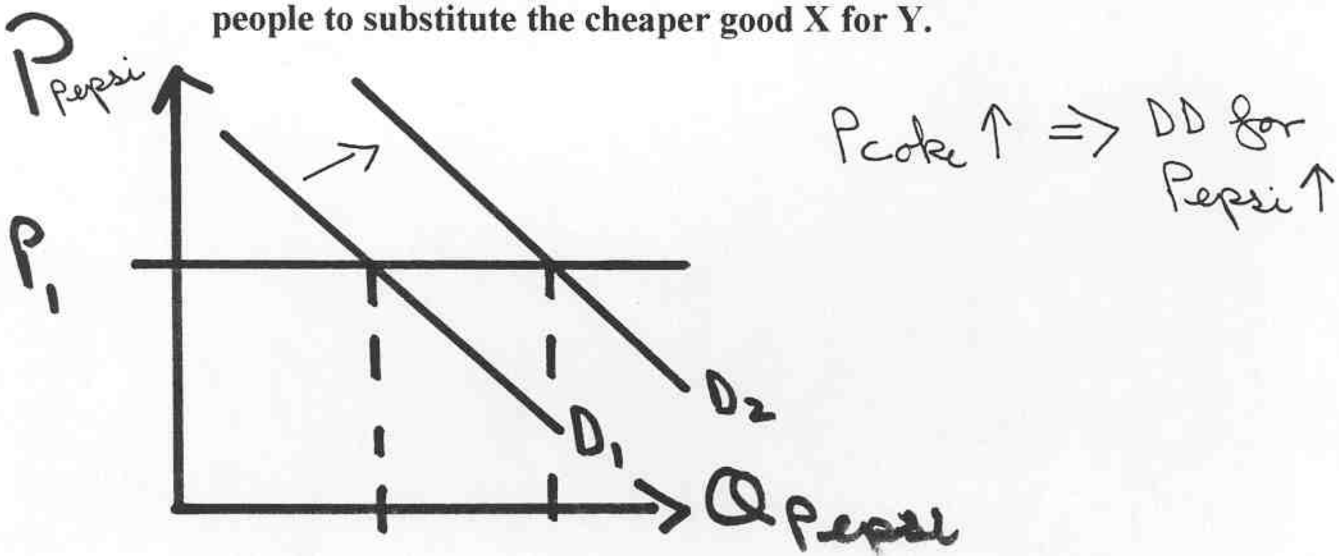
X= Demand for good X
P_Y = Price of good Y
ΔX = Change in demand for X
ΔP_y = Change in the price of Y

- **Substitutes Vs. Complements**

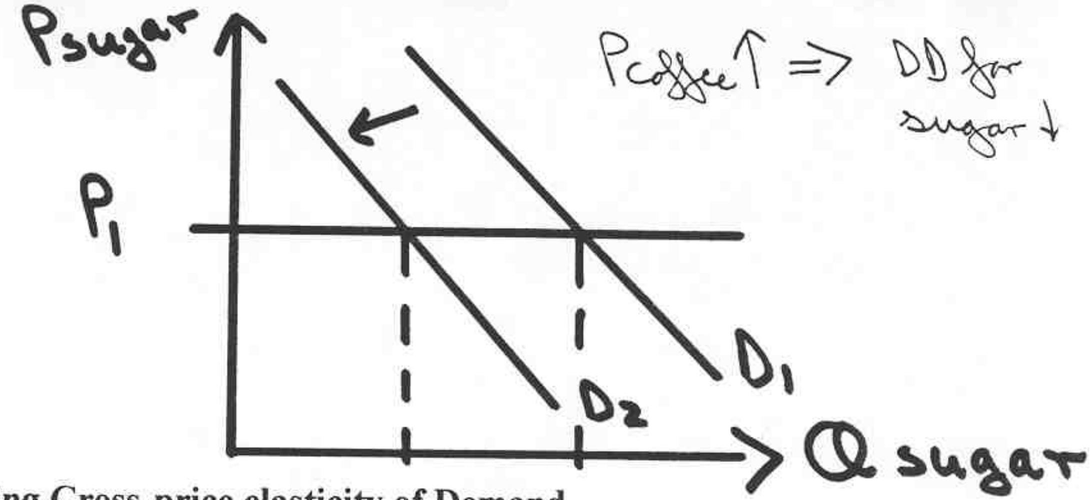
  - **Substitutes in consumption are goods that can be used in place of each other e.g. you can substitute Coke for Pepsi.**

  - **Complements are goods that are consumed together e.g. Coffee, cream & sugar.**

  - **The cross-price elasticity for substitutes in consumption is positive. This is because a rise in the price of good Y will cause people to substitute the cheaper good X for Y.**



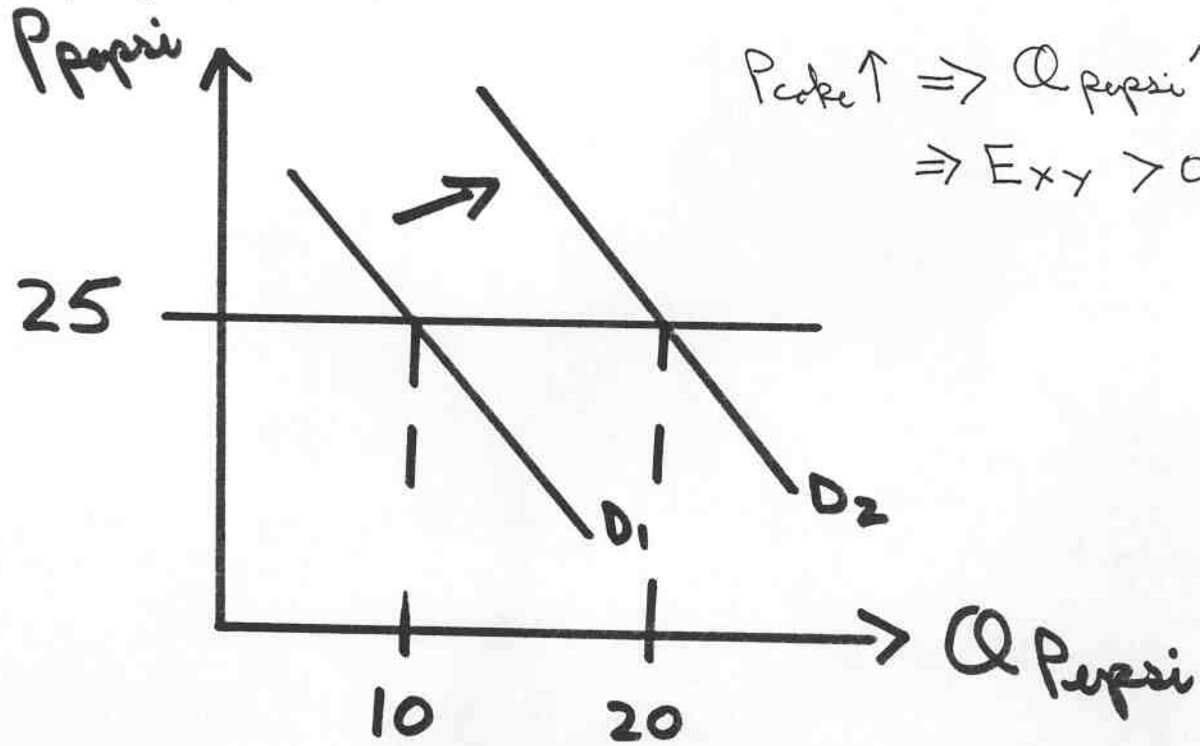
- **The cross-price elasticity for complements in consumption is negative. This is because a rise in the price of good Y will cause people to consume less of good Y. Since they consume X & Y together, they will consume less of good X as well.**



- **Calculating Cross-price elasticity of Demand**

Price of Pepsi per can	# of cans of Pepsi	Price of Coke per can
25	10	20
25	20	30

$$E_{XY} = \frac{\Delta X / X}{\Delta P_Y / P_Y} = \frac{(10-20)/15}{(20-30)/25} = \frac{-10/15}{-10/25} = \frac{5}{3} = 1.67$$



3. Cross-price Elasticity of Supply

- **Definition & Formula**

It is the proportional (percentage) change in the supply for good X divided by the proportional (percentage) change in the price of good Y

$$\text{Cross Price Elasticity of Supply} = \frac{\text{Percentage change in the Supply of X}}{\text{Percentage change in Price of Y}}$$

$$S_{XY} = \frac{\Delta X^S / X^S}{\Delta P_Y / P_Y} = \frac{\Delta X^S}{\Delta P_Y} * \frac{P_Y}{X^S}$$

$X^S =$ Supply of good X
$P_Y =$ Price of good Y
$\Delta X^S =$ Change in supply of X
$\Delta P_y =$ Change in the price of Y

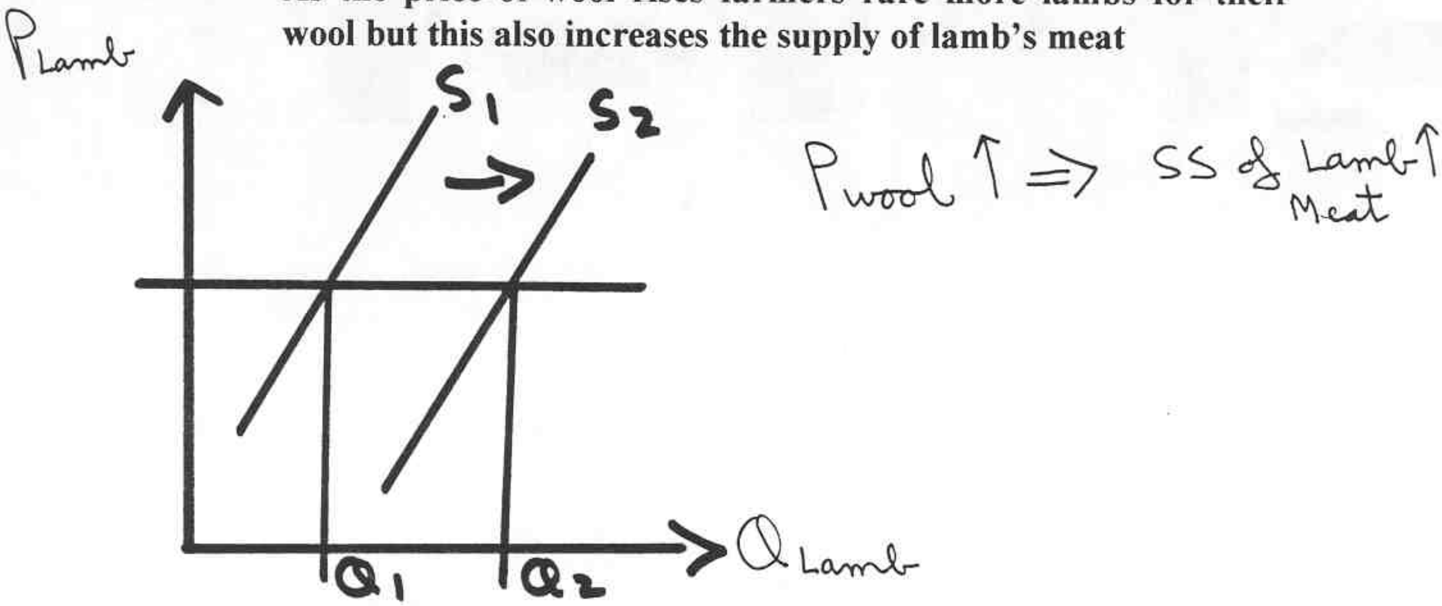
- **Joint Products & Substitutes in Production**

  - **Joint Products are those in which the production of one good yields the other good as a byproduct e.g. wool and lamb's meat.**

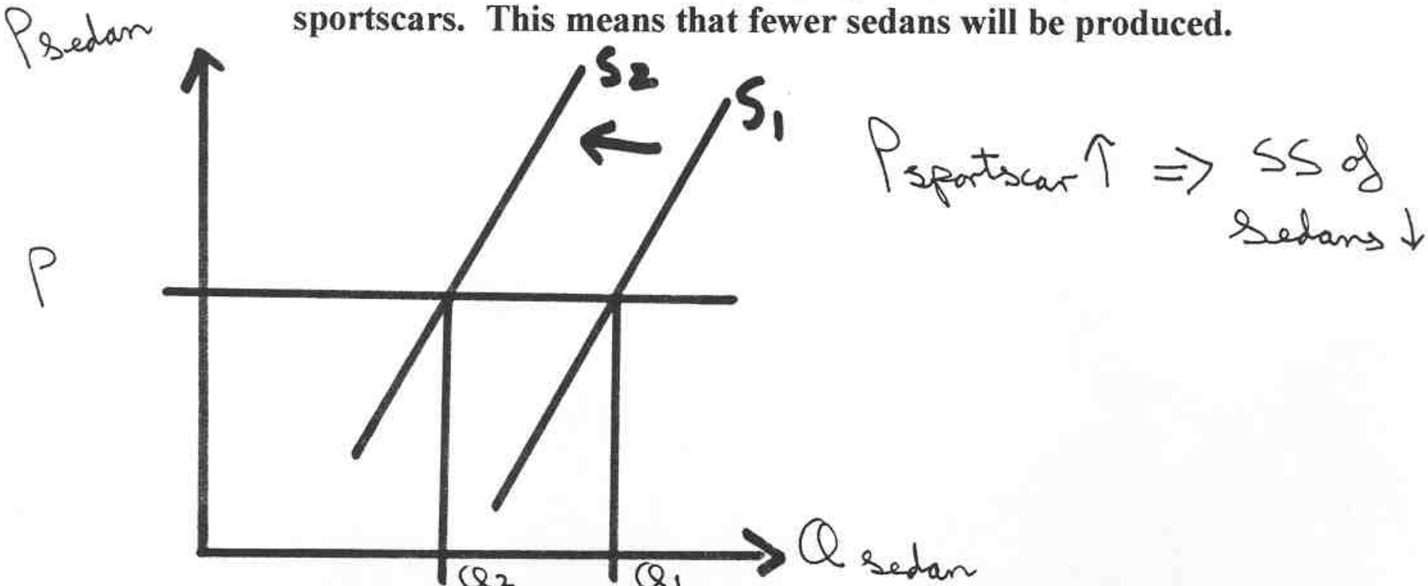
  - **Substitutes in production are those goods such that the production of one good reduces the quantity produced of the other good.**

    Example: A single assembly line that either produces sports cars or sedans.

- **Joint Products have a positive cross-price elasticity of supply. As the price of wool rises farmers rare more lambs for their wool but this also increases the supply of lamb's meat**



- **Substitutes in production have a negative cross-price elasticity of supply. As the price of sportscars goes up, the car manufacturer uses the assembly line to produce more sportscars. This means that fewer sedans will be produced.**



- **Calculating Cross-price elasticity of Supply**

Price of Sedans	# of Sedans Produced	Price of sportscars
25,000	10,000	20,000
25,000	5,000	30,000

4. Income Elasticity of Demand

- **Definition & Formula**

It is the proportional (percentage) change in the demand for good X divided by the proportional (percentage) change in income.

$$\text{Income Elasticity of Demand} = \frac{\text{Percentage change in the Demand for X}}{\text{Percentage change in Income}}$$

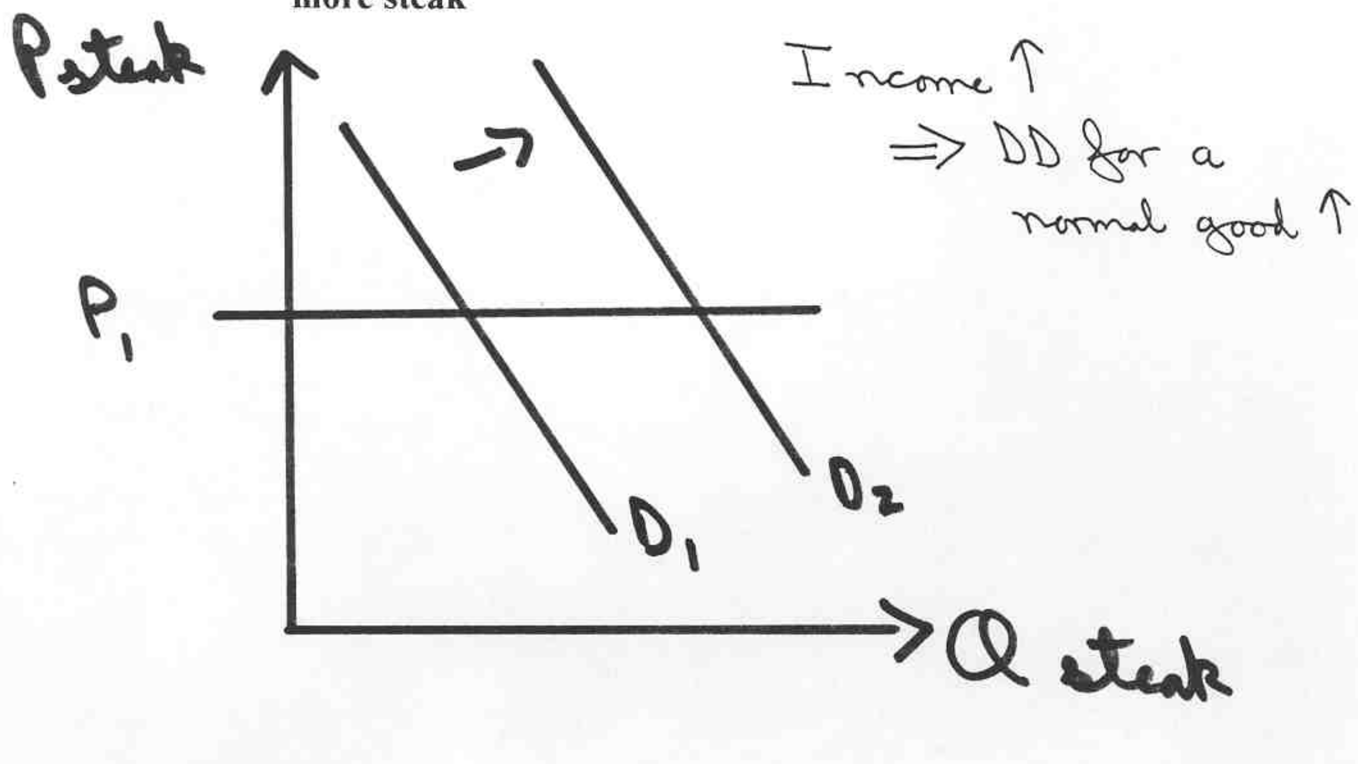
$$\eta = \frac{\Delta X / X}{\Delta I / I} = \frac{\Delta X}{\Delta I} * \frac{I}{X}$$

X= Demand for good X
I = Income
ΔX = Change in demand for X
Δ I = Change in Income

- **Normal Vs. Inferior Goods**

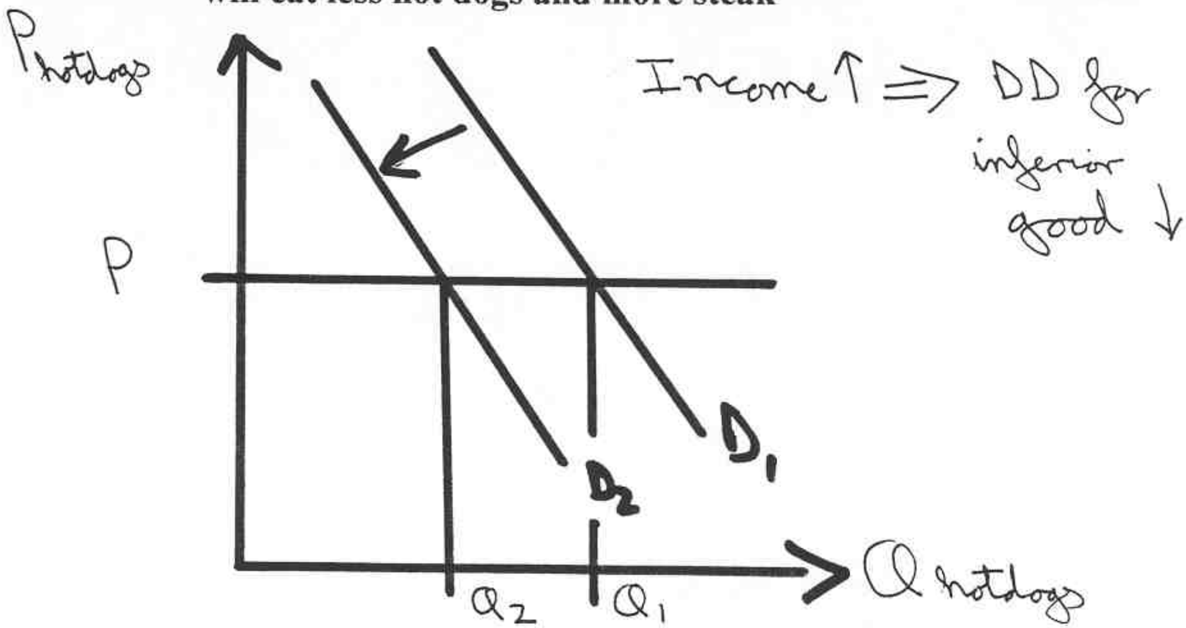
  - **A normal good is one whose demand increases with an increase in income. The income elasticity of a normal good is positive.**

    Example: After getting a raise in pay you may tend to eat more steak



- **An inferior good is one whose demanded is negatively related to income. The income elasticity of an inferior good is negative.**

  As you get done with school and earn more money you will eat less hot dogs and more steak



  - **Luxuries are goods with income elasticities greater than one. In other words they are goods for which the percentage change in demand is greater than the percentage change in income that caused that change.**

    Example: Given a 10% rise in income the change in demand for fishing rods is 20%. The income elasticity of fishing rods is 2. Would you consider them a luxury?

  - **Necessities are those good with an income elasticity less than one but greater than zero.**

    Example: Milk. It is unlikely that a rise in income will spur bouts of increased milk drinking.

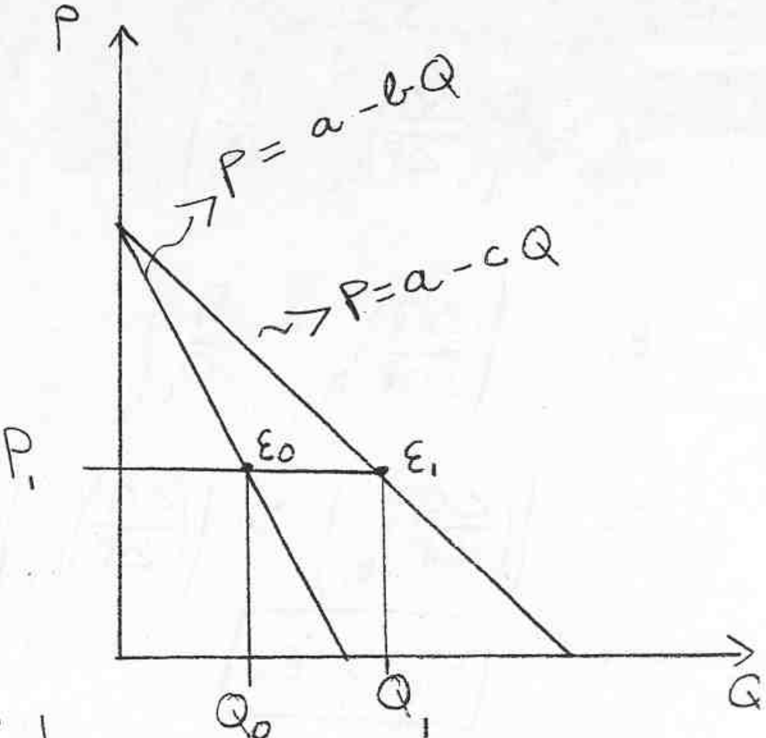
- **Determinants of Income Elasticity**

  - **How basic is the item in the consumption pattern?**

    EX: Milk Vs. Veal

Puzzlolutions

#1) $|\varepsilon_0| \gtrless |\varepsilon_1|$?



$$\varepsilon_0 = \left|\frac{\Delta Q}{\Delta P} * \frac{P}{Q}\right| = \left|-\frac{1}{b} * \frac{P_1}{Q_0}\right|$$

$$= \left|-\frac{1}{b} * \frac{(a - bQ_0)}{Q_0}\right|$$

$$\varepsilon_0 = \frac{a - bQ_0}{bQ_0} \quad (1)$$

$$\varepsilon_1 = \left|\frac{\Delta Q}{\Delta P} * \frac{P}{Q}\right| = \left|-\frac{1}{c} * \frac{P_1}{Q_1}\right|$$

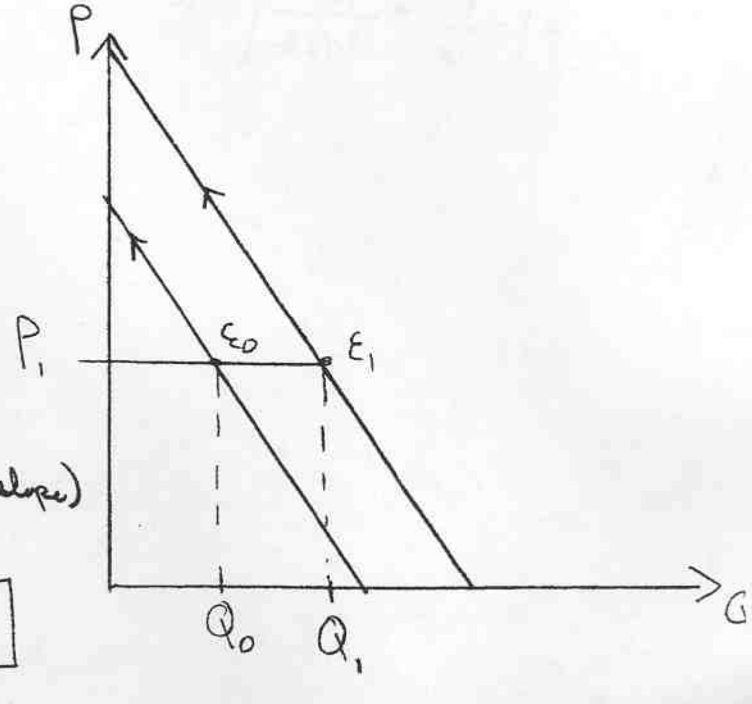
$$= \left|-\frac{1}{c} * \frac{(a - cQ_1)}{Q_1}\right| = \frac{a - cQ_1}{cQ_1} \quad (2)$$

$$\because P_1 = a - bQ_0 = a - cQ_1 \quad (3)$$

$$\therefore bQ_0 = cQ_1 \quad (4)$$

$$(1), (2), (3) + (4) \Rightarrow \varepsilon_0 = \varepsilon_1$$

#2) $|\varepsilon_0| \gtrless |\varepsilon_1|$?



$$\varepsilon_0 = \left|\frac{\Delta Q}{\Delta P} * \frac{P_1}{Q_0}\right|$$

$$\varepsilon_1 = \left|\frac{\Delta Q}{\Delta P} * \frac{P_1}{Q_1}\right|$$

$\because \frac{\Delta Q}{\Delta P} = \frac{\Delta Q}{\Delta P}$ (parallel lines, same slope)

$$\because \frac{P_1}{Q_0} > \frac{P_1}{Q_1} \quad \therefore \boxed{\varepsilon_0 > \varepsilon_1}$$

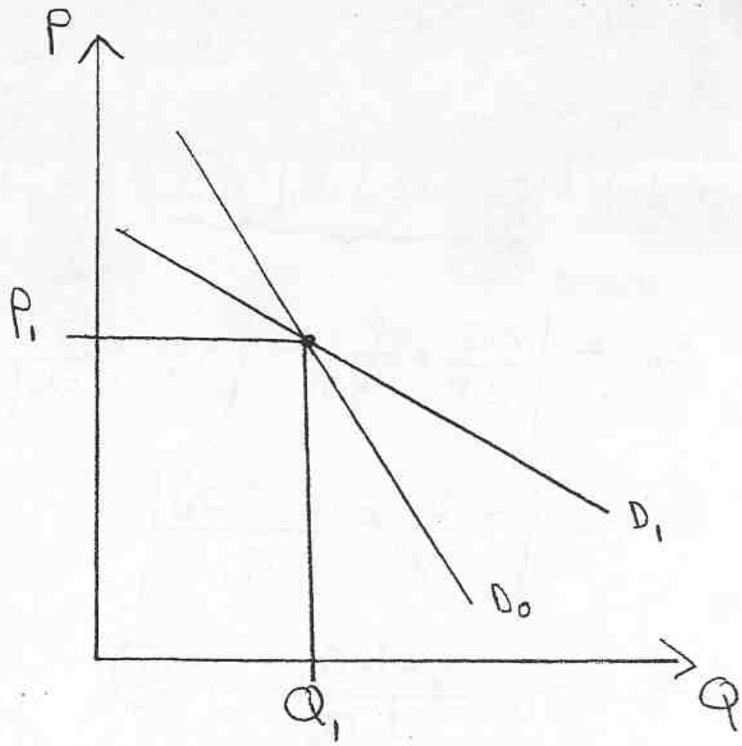
(#3) $|\varepsilon_0| \gtrless |\varepsilon_1|$?

$$\varepsilon_0 = \left| \left(\frac{\Delta Q}{\Delta P}\right)_{D_0} * \frac{P_1}{Q_1} \right|$$

$$\varepsilon_1 = \left| \left(\frac{\Delta Q}{\Delta P}\right)_{D_1} * \frac{P_1}{Q_1} \right|$$

$$\because \left| \left(\frac{\Delta Q}{\Delta P}\right)_{D_1} \right| > \left| \left(\frac{\Delta Q}{\Delta P}\right)_{D_0} \right|$$

$$\therefore \boxed{\varepsilon_1 > \varepsilon_0}$$



(#4) Show that $|\varepsilon| = 1$ at the midpoint of a straight line demand function.

$$|\varepsilon| = \left| \frac{\Delta Q}{\Delta P} * \frac{P_1}{Q_1} \right|$$

$$= \left| -\frac{1}{b} * \frac{a/2}{a/2b} \right| = 1$$