# Calculus II Laboratory: Taylor Polynomials

This is a hand-in laboratory, and should be done with a partner (you and your partner should hand in one lab report). Your lab report should be a narrative of what you did, and the conclusions that you drew, following the numbered outline below. Please use complete sentences and good English usage. Your report should be self-contained and readable without reference to this handout. Feel free to consult your textbook, your notes and me about the problems in this lab. You may speak to anybody about technical *DERIVE* questions.

In this lab you will be exploring approximations of the cosine function using various Taylor polynomials.

1. Find the Taylor polynomial $p_{10}(x)$ for $\cos x$, centered at $a = 0$. Examine the graphs of $\cos x$ and $p_{10}(x)$. Use the graph to describe how well $p_{10}(x)$ approximates $\cos x$ for $x = 1, 3, 5$.

2. For a quantitatively more precise analysis, let's define the *relative error function* as
$$E(x) = \frac{\cos x - p_{10}(x)}{\cos x}.$$
This is useful, because if we take the absolute value of $E(x)$ and multiply by 100, this give the error *as a percentage of the size of the function.* By this standard, how good are the approximations for $x = 1, 3, 5$?

3. Now consider the eighteenth degree Taylor polynomial $p_{18}(x)$. Plot the relative error function for this estimate, and discuss its accuracy on the interval $[-\pi, \pi]$.

4. The cosine function is periodic, and so $\cos(10) = \cos(10 - 4\pi)$. Use this fact to obtain an accurate approximation of $\cos(10)$ using $p_{18}(x)$. Calculate the relative error of this approximation.

5. Use the same idea as in the previous part to compute an approximation for $\cos(2438762)$.

6. We could instead use a Taylor polynomial centered at a different point to estimate $\cos(10)$. Find the sixth degree Taylor polynomial for $\cos x$ centered at the point $3\pi$. Use this Taylor polynomial to approximate $\cos(10)$. Compare the relative error here to that you found above.