Calculus II Laboratory: Approximating Definite Integrals

This is a hand-in laboratory, and should be done with a partner (you and your partner should hand in one lab report). Your lab report should be a narrative of what you did, and the conclusions that you drew, following the numbered outline below. Please use complete sentences and good English usage. Your report should be self-contained and readable without reference to this handout. Feel free to consult your textbook, your notes and me about the problems in this lab. You may speak to anybody about technical *DERIVE* questions.

At a point near a drainage ditch, a scientist models the *rate at which the water table falls.* Her function is

$$v(t) = \frac{10e^{-\frac{1}{t+0.1}}}{(t+0.1)^{\frac{3}{2}}},$$

where t is measured in days, and $v(t)$ is measured in *inches per day.*

1. Explain what our scientist is computing, if she computes the integral $\int_0^3 v(t)dt$. (Think about the fundamental theorem of calculus!)
2. What happens if you ask *DERIVE* to compute this integral exactly? (ERF is a function we don't know!) What does this tell you about our chance of doing this integral by hand? What value do you get if you approximate this integral with *DERIVE*?
3. Let's now do our own integral approximations. We will begin with 100 subintervals. Then $\Delta t = 3/100$. Using sigma notation, write down by hand the expressions for L_{100} and R_{100}. Then implement these in *DERIVE*, using the *Sum* command under the *Calculus* menu. Record your results. What is the approximation T_{100}?
4. Look at the graph of $v(t)$. Can you tell whether your approximations are overestimates or underestimates, by just looking at the graph?
5. Determine the high point on the curve, which is of course where it changes from being an increasing function to being a decreasing function – describe how you did this.
6. Now break up your integral into two pieces, and obtain approximations using left or right summations with 100 terms, which are guaranteed to be either underestimates or overestimates.