5] (a) $1 = \int |f|^2 \sin\theta \, d\theta \, d\varphi = \int_0^{2\pi}\int_0^{\pi} |A|^2 \sin^4\theta (7\cos^2\theta - 1)^2 \sin\theta \, d\theta \, d\varphi$

$= 2\pi |A|^2 \int_0^{\pi} \sin^5\theta (7\cos^2\theta - 1)^2 d\theta = 2\pi |A|^2 \frac{64}{45}$

$\Rightarrow A = \sqrt{\frac{1}{2\pi}\frac{45}{64}} = \frac{3}{8}\sqrt{\frac{5}{2\pi}}$

(b) f is an angular momentum eigenfunction iff $L^2 f = bf$ where b is a constant (specifically, $b = l(l+1)\hbar^2$)

$L^2 = -\hbar^2\left(\frac{\partial^2}{\partial\theta^2} + \cot\theta \frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right)$

$\frac{\partial f}{\partial\theta} = \left[2A \sin\theta \cos\theta (7\cos^2\theta - 1) - 14A \sin^3\theta \cos\theta\right] e^{im\varphi}$

$\frac{\partial^2 f}{\partial\theta^2} = \left[2A(\cos^2\theta - \sin^2\theta)(7\cos^2\theta - 1) - 70A\sin^2\theta\cos^2\theta + 14A\sin^4\theta\right] e^{im\varphi}$

$\frac{\partial^2 f}{\partial\varphi^2} = -m^2 A \sin^2\theta (7\cos^2\theta - 1) e^{im\varphi}$

$L^2 f = -\hbar^2 A \Big\{ 2(\cos^2\theta - \sin^2\theta)(7\cos^2\theta - 1) - 70\sin^2\theta\cos^2\theta + 14\sin^4\theta$

$+ \frac{\cos\theta}{\cancel{\sin\theta}}\left[2\cancel{\sin\theta}\cos\theta(7\cos^2 - 1) - 14\sin^{\cancel{3}\,2}\theta\cos\theta\right]$

$+ \frac{1}{\cancel{\sin^2\theta}}\left[-m^2 \cancel{\sin^2\theta}(7\cos^2\theta - 1)\right]\Big\} e^{im\varphi}$

$= -\hbar^2 A e^{im\varphi}\left[2(2\cos^2\theta - \sin^2\theta - m^2)(7\cos^2\theta - 1) - 84\sin^2\theta\cos^2\theta + 14\sin^4\theta\right]$

$= -\hbar^2 \underbrace{A e^{im\varphi}\sin^2\theta(7\cos^2\theta - 1)}_{f(\theta,\varphi)}\left[\frac{2(2\cos^2\theta - \sin^2\theta - m^2)}{\sin^2\theta} + \frac{-84\cos^2\theta + 14\sin^2\theta}{(7\cos^2\theta - 1)}\right]$

$= -\hbar^2 f\left[\frac{2(2(1-\sin^2\theta) - \sin^2\theta - m^2)}{\sin^2\theta} + \frac{-84\cos^2\theta + 14(1-\cos^2\theta)}{(7\cos^2\theta - 1)}\right]$

$= -\hbar^2 f\left[\frac{4 - m^2}{\sin^2\theta} - 6\frac{\cancel{\sin^2\theta}}{\cancel{\sin^2\theta}} + \frac{-98\cos^2\theta + 14}{7\cos^2\theta - 1}\right]$

$= -\hbar^2 f\left[\frac{4-m^2}{\sin^2\theta} - 6 - 14\frac{\cancel{(7\cos^2\theta - 1)}}{\cancel{7\cos^2\theta - 1}}\right]$

$= -\hbar^2 f\left[\underbrace{\frac{4-m^2}{\sin^2\theta}} - 20\right]$

This term is zero iff $\boxed{m = \pm 2}$

$= +20\hbar^2 f = l(l+1)\hbar^2 f$ iff $\boxed{l = 4}$

so f is an eigenfunction of L^2 iff $l = 4$, $m = \pm 2$