3) $\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$ $\quad\quad$ $\Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2}$

For all energy eigenstates, $\langle x\rangle = \frac{L}{2}$ $\quad \langle p\rangle = 0 \quad \langle p^2\rangle = 2m\langle E\rangle$

$\rightarrow \langle p^2\rangle = \frac{\pi^2\hbar^2}{L^2} n^2$

so, all you need to calculate is

$\langle x^2\rangle = \int x^2 u_n^2(x)\,dx \quad\quad u_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$

$$\sin^2 A = -\frac{1}{4}\left(e^{i2A} + e^{-i2A} - 2\right) = \frac{1}{2} - \frac{1}{2}\cos 2A$$

$$\langle x^2\rangle = \frac{2}{L}\int_0^L x^2 \sin^2\left(\frac{n\pi x}{L}\right)dx = \frac{1}{L}\int\left(x^2 - x^2\cos\left(\frac{2n\pi x}{L}\right)\right)dx$$

$$= \frac{1}{L}\left\{\frac{1}{3}x^3 - \frac{Lx^2}{2n\pi}\sin\left(\frac{2n\pi x}{L}\right) - \frac{L^2 x}{2n^2\pi^2}\cos\left(\frac{2n\pi x}{L}\right) + \frac{L^3}{4n^3\pi^3}\sin\left(\frac{2n\pi x}{L}\right)\right\}\Bigg|$$

$$\langle x^2\rangle = \frac{1}{L}\left\{\frac{1}{3}L^3 + \frac{L^3}{2n^2\pi^2}\right\} = L^2\left(\frac{1}{3} - \frac{1}{2n^2\pi^2}\right)$$

$$\text{So } \Delta p = \sqrt{\frac{\pi^2\hbar^2}{L^2}n^2 - 0} = \frac{n\pi\hbar}{L}$$

$$\Delta x = \sqrt{L^2\left(\frac{1}{3} - \frac{1}{2n^2\pi^2} - \frac{1}{4}\right)} = L\sqrt{\frac{1}{12} - \frac{1}{2n^2\pi^2}}$$

$\Delta x \Delta p = n\pi\hbar\sqrt{\frac{1}{12} - \frac{1}{2n^2\pi^2}}$ $\quad\quad$ If $n=1$ $\Delta x\Delta p = .568\hbar$

If $n=2$ $\Delta x\Delta p = 1.67\hbar$

The uncertainty is smaller for $n=1$, but just above the minimum value of $\hbar/2$.

As n gets large $\Delta x \rightarrow L/\sqrt{12}$ while $\Delta p = n\pi\hbar/L$

so

$\Delta x\Delta p \rightarrow \frac{n\pi\hbar}{\sqrt{12}}$ $\quad$ which just increases with n.