MA 110/ 127 Final Topics.

Information about the Final: The Final will be available on Wednesday morning at 8:30 a.m. and will be due by noon. It will be designed to take about two hours.

The honor code will apply as follows. The test is to be taken in the library mezzanine area (2nd floor) or in the classroom or in the math lounge area. Please familiarize yourself with the honor code and don't talk about problems, even to say which problems are hard/easy until you/your friends have finished the test.

Calcuators will be allowed, and you will be asked to show all your work. You may make a 3 x 5 " index card of notes (both sides) but you are asked to use a font size of at least four.

Plan to refer to door of my office, Tutt 206D, for any hints or corrections during the course of the test, before you turn it in. Also, I will be in my office from 8:30 a.m. to 9:45 a.m. and from 10:30 a.m. to noon. (I may step out between 9:45 to 10:30 for brief periods).

Below is a list of topics that may be on the test:

Calculus

Limits:

Be able to find limits using methods we discussed in class (possibly without L'Hospital's rule).

Be able to determine if a limit exists or not.

Derivatives:

- Find derivatives of functions including arcsin, arctan, and using logarithmic differentiation.
- Be able to explain how to find derivatives of inverse functions using implicit differentiation
- Use the following or combinations of the product rule, quotient rule and chain rule.
- Be able to calculate the derivative from the defintion using a limit e.g.

$$\begin{aligned}\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h} &= \lim_{h\to 0}\frac{\frac{x}{x(x+h)}-\frac{x+h}{x(x+h)}}{h}\\ &= \lim_{h\to 0}\frac{\frac{x-(x+h)}{x(x+h)}}{h}\\ &= \lim_{h\to 0}\frac{-h}{x\left(x+h\right)}\frac{1}{h}\\ &= \lim_{h\to 0}\frac{-1}{x\left(x+h\right)}=\frac{-1}{x\left(x+0\right)}=\frac{-1}{x^2}\end{aligned}$$

- Be able to use the definition of derivative to determine if the derivative exists or not (just as a limit can exist or not).
- Know that a differentiable function must be continuous.
- Know how to differentiate implicitly.

Newton's Method:

- Know what a root of a function is.
- Know what a basin of attraction for a root is
- Be able to iterate using Newton's method.

Using the Derivative:

- Know the derivative tests.
- Use f or f' or f'' to determine information about f such as
  - where is f increasing and where is it decreasing.
  - concavity of f
- Know the mean value theorem
- Apply intermediate value theorem to determine if there is a solution to certain equations.
- There won't be a graph sketching question on the test.
- Optimization

Integral Calculus:

Vocabulary: upper (lower) limit of integration, Riemann sum, definite integral, indefinite integral, upper (lower) limit of summation, Δx, x_i, sample points x_i^*, integrand, left endpoint, midpoint and right endpoint rules.

- Be able to draw in rectangles for a given n and rule for choosing x_i^*.
- Know the relationship of velocity and position via the integral.
- Be able to evaluate definite integrals using the definition of integral (i.e. using the long Riemann sum definition).
- Be able to pronounce Leibniz correctly.
- Given a rate of change, calculate the total change of the quantity over a given interval.
- Know the difference between the definite integral and the indefinite integral.
- Find antiderivatives using the substitution rule.

Applications of integration

Find areas between curves, possibly integrating dy

Find volumes by the method of cross sections, which may or may not be a solid of revolution

Find volumes by the method of cylindrical shells

Chaos and Iteration

Iteration:

- Be able to find fixed points for a function, either graphically or numerically
- Be able to determine fixed points for the tent map and the logistic map
- Be able to determine if there is a 2 cycle for the tent map and for the logistic map
- Be able to to read information from bifurcation diagrams to determine facts about the dynamics for a given value of the control parameter..
- Be able to identify the number of n-cycles of a function f given appropriate information about the return maps of f.
- Be able to recognize sensitivity to initial conditions.
- Be able to explain period doubling and the Feigenbaum constant

Theorems from Chaos Theory to apply

Theorem: If you iterate a continuous function f to make a sequence $\{x_0, x_1, ...\}$ and it converges then it must be a fixed point of f.

Theorem: If f is continuous and I and J are intervals and $f(I) \supseteq J$ then f has a fixed point in the interval I.

Theorem: In Newton's method, if a sequence of Newton's method iterates converges then the limit is a root of the function

Theorem: If there is a 3-cycle for a continuous function, then there is an n-cycle for any n.